PHYSICS

11th

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A Textbook of Physics
for Grade 11

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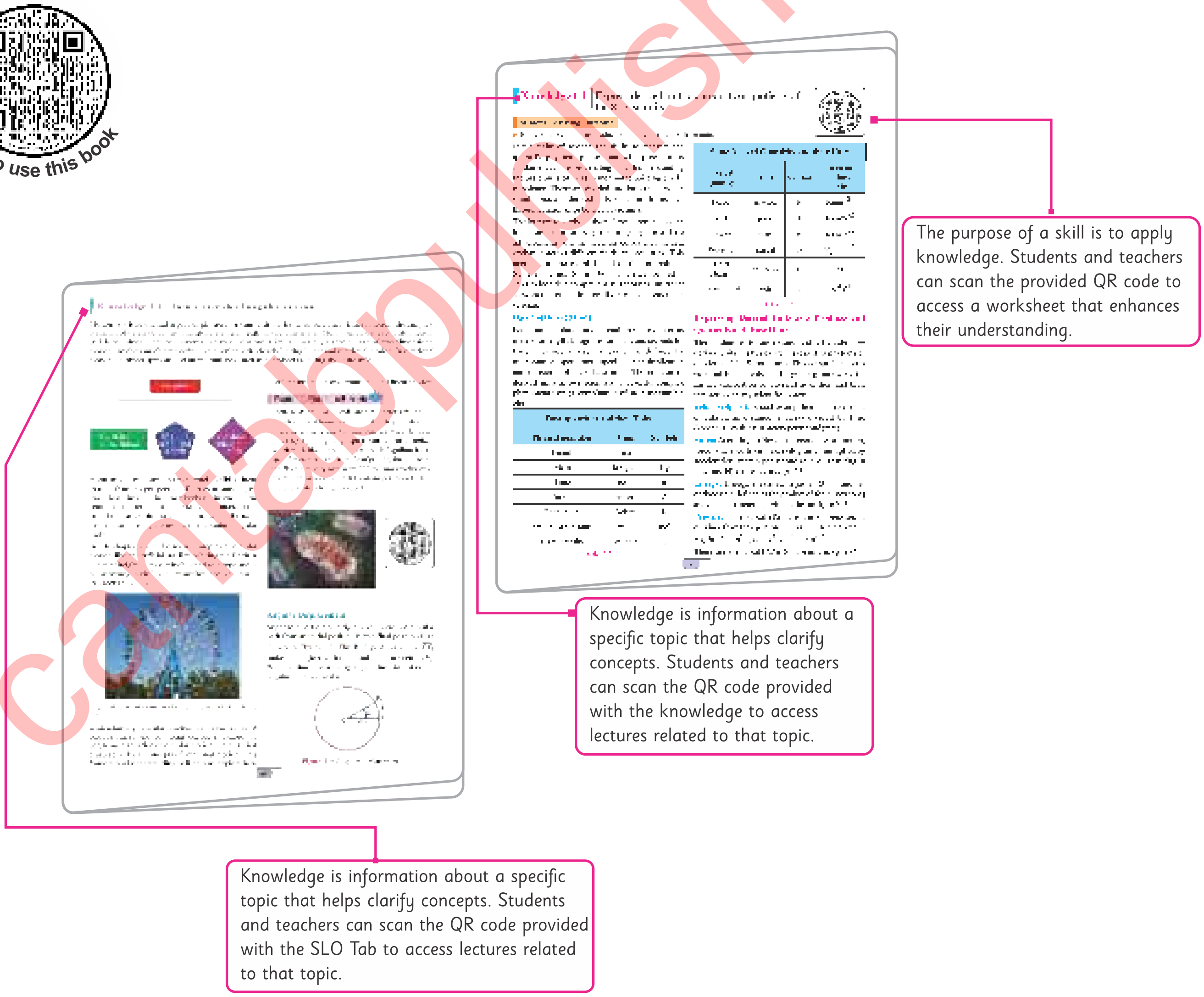
Preface

This advanced Grade 11 Physics textbook marks a significant advancement in educational resources, utilizing the Concrete, Pictorial, Abstract (CPA) Approach. This method begins with hands-on examples, progresses through visual representations, and ultimately reaches abstract concepts, catering to different learning styles. This approach makes the subject matter both accessible and engaging for students.The textbook vividly brings to life the principles of physics through interactive visuals and real-life applications. This method transforms complex theories into understandable, relatable concepts, enhancing comprehension and sparking student interest.A notable feature of the book is its interactive "Test Yourself" sections, which encourage active participation and self-evaluation. The included classroom activities are designed to promote teamwork and critical thinking, enriching the educational experience. "Teacher's Footnotes" are also provided, offering valuable insights for more effective instruction.The textbook is visually appealing, with interactive color illustrations that create a vibrant and captivating learning environment, moving beyond traditional textbook formats. It includes a diverse array of examples, worksheets, and video lectures for a well-rounded educational experience. Additionally, the book incorporates simulations for an interactive approach to understanding physics concepts.More than just a teaching aid, this textbook is a journey through the world of physics. It aligns the subject with modern educational trends, making it an essential tool for both instruction and learning in the contemporary educational context of Grade 11.

How to use this book



SLO Based Model Video Lecture

Salient Features

Comprehensive Learning
Engage students with videos, simulations, and practical worksheets.

Structured Lesson Plan
Well-organized with clear objectives, PPTs, and a self-assessment questions.

Engaging Multimedia
Visual appeal through PPTs and interactive simulations.

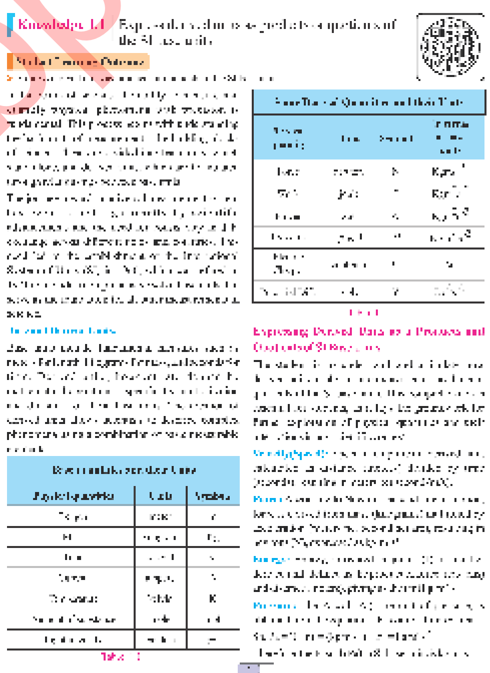
Assessment & Tracking
Diverse question from daily life and progress monitoring.

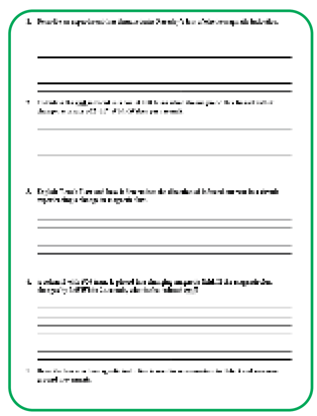
Adaptable & Accessible
Scalable and accessible, suitable for all learners.

SLO No: P: 09 - E -41
SLO statement: Explain experiments that demonstrate Faraday's and Lenz's laws

Simulation

KNOWLEDGE



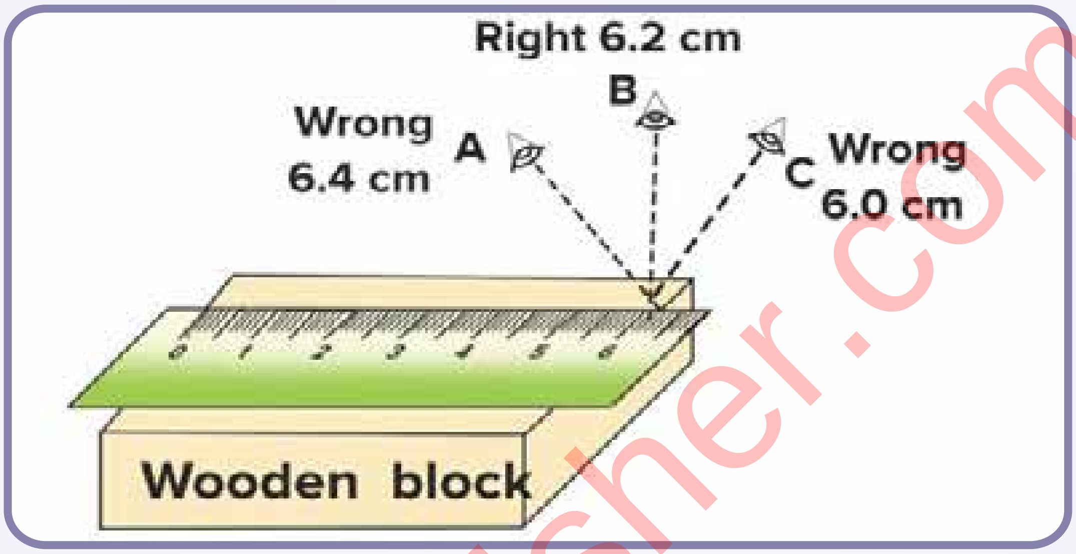


Skills Sheet

CONTENTS

CHAPTER 1

Measurements



Student Learning Outcomes

Knowledge 1.1

Express derived units as products or quotients of the SI base units

[SLO: P-11-A-02]: Express derived units as products or quotients of the SI base units.

Knowledge 1.2

Making Reasonable Estimates of Physical Quantities

[SLO: P-11-A-01]: Make reasonable estimates of physical quantities. [Of those quantities that are discussed in the topics of this grade] .[Including that these consist of a magnitude and a unit]

Knowledge 1.3

Dimensional Analysis

[SLO: P-11-A-03]: Analyze the homogeneity of physical equations [Through dimensional analysis]

[SLO: P-11-A-04]: Derive formulae in simple cases. [Through using dimensional analysis]

Knowledge 1.4

Uncertainties in Measurement:

[SLO: P-11-A-06]: Assess the uncertainty in a derived quantity. [By simple addition of absolute, fractional or percentage uncertainties]

[SLO: P-11-A-07]: Justify why all measurements contain some uncertainty.

Knowledge 1.5

Analyzing and Critiquing the Accuracy and Precision

[SLO: P-11-A-05]: Analyse and critique the accuracy and precision of data collected by measuring instruments

Introduction

In this chapter, students will explore the basics of measuring and analyzing physical quantities in physics. They will learn how to make estimates of these quantities, understanding that each has a magnitude and a unit. The chapter will also teach students how to express derived units using the basic SI units and how to check if physical equations are consistent using dimensional analysis. Additionally, students will learn to derive simple formulas and analyze the accuracy and precision of data from measuring instruments. The chapter will also cover how to assess uncertainties in measurements and explain why all measurements have some level of uncertainty. These concepts are essential for conducting accurate scientific experiments.

Knowledge 1.1 | Express derived units as products or quotients of the SI base units

In the realm of science, the ability to measure and quantify physical phenomena with precision is fundamental. This process begins with understanding of the basic units of measurement—the building blocks of science. These are divided into base units, which have independent identification and derived units, which are formulated from relationships between base units.

The need for a universal measurement system has been greatly influenced by scientific progress and the necessity for consistent data exchange across various fields and countries. This led to the creation of the International System of Units (SI) in 1960. In 1971, it was improved to include a complete set of base units, which form the basis for all other scientific measurements.

Base and Derived Units:

Base units include fundamental measures such as meter for length, kilogram for mass, and second for time as given in Table 1.1. Derived units, however, are defined by mathematical operations—specifically multiplication and division—of these base units. The formation of derived units allows scientists to describe complex phenomena.

Table: 1.1: Base quantities and their Units

Physical quantities	Units	Symbols
Length	meter	m
Mass	kilogram	kg
Time	second	s
Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Light intensity	candela	cd

Table: 1.2: Some Derived Quantities and their Units

Physical quantity	Unit	Symbol	In terms of base units
Force	newton	N	$kg\ m\ s^{-2}$
Work	joule	J	$kg\ m^2\ s^{-2}$
Power	watt	W	$kg\ m^2 s^{-3}$
Pressure	pascal	Pa	$kg\ m^{-1} s^{-2}$
Electric Charge	coulomb	C	As
Potential Difference.	volt	V	$kg\ m^2 s^{-3} A^{-1}$

Expressing Derived Units as a Products and Quotients of SI Base Units

Students must understand and explain how derived units in physics are expressed as combinations (products or quotients) of the SI base units. This understanding is essential as it forms the foundation for exploring physical quantities and their interactions in any scientific context.

Velocity(Speed): Speed exemplifies a derived unit, calculated as distance (meters) divided by time (seconds), resulting in meters per second (m/s).

Force: According to Newton's second law of motion, force is derived from mass and acceleration as

Force: = mass ×aceleration

= mass × (change in velocity) / (time)

Now the unit of force (N) is given as

$$N = kg \times \frac{m}{s^2} = kg\ m\ s^{-2}$$

Energy: Energy, measured in joules (J), is another derived unit, defined as the product of force (newtons) and distance (meters) i.e.,

Energy = force ×distance

So the unit of energy is $J = kg\ m\ s^{-2} \times m = kg\ m^2\ s^{-2}$

Pressure: The Pascal (Pa), the unit of pressure, is obtained from the equation: $P = \frac{F}{A}$

So, $Pa = \frac{N}{m^2} = \frac{(kg\ m\ s^{-2})}{m^2} = kg\ m^{-1} s^{-2}$

Therefore, the Pascal (Pa) in SI base units is $kg\ m^{-1}\ s^{-2}$

Electric Charge (Coulomb, C): Expressed as ampere times second (As).

Electric Potential (Volt, V): It is the potential energy per unit charge defined as Joule per Coulomb (J/C), which further breaks down to $(kg\ m^2 s^{-2}) / (A\ s)$ $= kg\ m^2\ s^{-3}\ A^{-1}$.Some other derived quantities with their derived units are listed in the Table 1.3

Table 1.3: Derived Units as Product and Quotient of SI Base Units

Derived Quantity	Description	Derived Unit (SI)	Expressed as Product/Quotient of SI Base Units	Expressed in Base Units
Area	Measure of a surface	square meter (m^2)	(Length × Width) $m \times m$	m^2
Volume	Measure of space occupied	cubic meter (m^3)	(Length × Width × Height) $m \times m \times m$	m^3
Power	Rate of doing work	watt (W)	(Energy / Time) $J\ s^{-1}$	$kg\ m^2\ s^{-3}$
Electric Potential	Electric potential energy per charge	volt (V)	(Power / Electric Current) $W\ A^{-1}$	$kg\ m^2\ s^{-3}\ A^{-1}$
Electric Resistance	Opposition to the flow of electric current	ohm (Ω)	(Electric Potential/ Electric Current) $V\ A^{-1}$	$kg\ m^2 s^{-3}\ A^{-2}$
Magnetic Flux	Measure of the strength of a magnetic field	weber (wb)	(magnetic field . area) Tm^2	$kg\ m^2\ s^{-2}\ A^{-1}$
Magnetic Flux Density	Measure of magnetic field strength	tesla (T)	(Magnetic Flux / Area) $Wb\ m^{-2}$	$kg\ s^{-2}\ A^{-1}$
Frequency	Number of occurrences per unit time	hertz (Hz)	$\left(\frac{1}{\text{Time Period}}\right)$ s^{-1}	s^{-1}

Skill 1.1

- Proficiency in understanding and using SI base units to express complex derived units, enhancing clarity in scientific calculations and communication.

Do you Know?

SI units provide a universal standard for measurements, ensuring that scientific and technical data is consistent and comparable worldwide, which is essential for global collaboration, technological development, and scientific research.

Knowledge 1.2 Making Reasonable Estimates of Physical Quantities

In a Grade 9, the fundamental skill of estimating physical quantities have been developed. It involves making an educated guess about the size, amount, or extent of a physical quantity, when precise measurements are not available. This skill is not just about guessing; it is about applying scientific understanding and reasoning to predict a quantity's approximate value. This skill is important because in real life and in other areas of science, you often cannot get exact measurements. Estimation lets you make quick, smart decisions. For example, when you want to buy a curtain, you do not need to know the exact size of the window, just a rough idea. Estimation also helps you get a feel for how big or small things are, like figuring out how far away a thunderstorm is by counting the time between lightning and thunder.

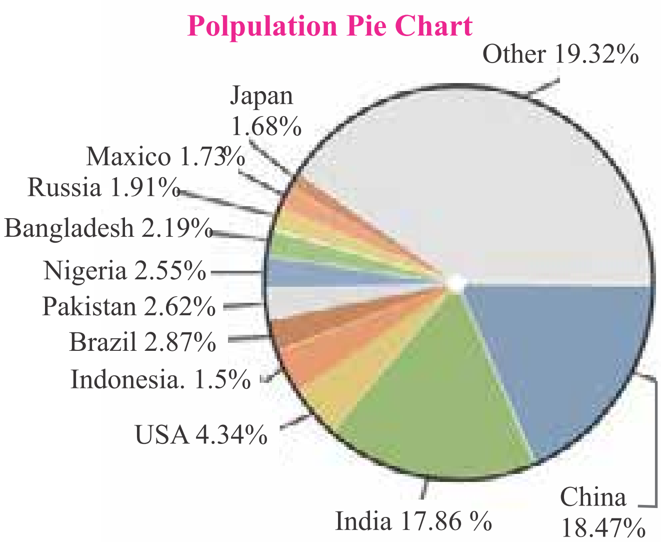


Fig 1.1: Estimation of Population of Different Countries

The pie chart in Fig. 1.1 illustrates the estimated population distribution among various countries. It shows that China and India have the largest populations, with 18.47% and 17.86% respectively, while other countries like the USA, Indonesia, and Brazil have smaller percentages. This estimation helps to visualize and compare the relative sizes of populations without exact numbers, emphasizing the proportions rather than precise values.

Important Information

Estimating a physical quantity involves making an educated guess based on observations or prior knowledge, without using precise instruments. In contrast, measurement taken from an instrument like rulers, scales, or thermometers provides an accurate and exact value . Measurement ensures consistency and precision in scientific data.

How to Make Good Estimate

1. When estimating physical quantities, start by clearly identifying what needs to be measured, such as length, volume, or time.
2. To provide base of your estimates in reality, use familiar and appropriate reference points. For instance, the height of a known doorway can provide a practical baseline for estimating the height of a classroom.
3. For more complex measurements, break them down into smaller, manageable components, making the task easy and more accurate.
4. It is essential to integrate relevant physics principles into your estimates; for example, consider gravitational acceleration when calculating how fast an object falls.
5. Instead of a single number, provide a range within which you believe the quantity lies. This acknowledges the intrinsic uncertainty in the process.
6. Lastly, cross-validate your estimates by using multiple methods; for example, verify the volume of a pool by measuring both; its physical dimensions and the water capacity it holds. This comprehensive approach not only increases the accuracy of your estimates but also reinforces their validity through diverse methods of confirmation.

Imagine you want to estimate how long it will take to fill a swimming pool with a garden hose. First, understand the quantity - you are estimating time. Break down the problem: estimate the pool's volume (say, 10 meters long, 5 meters wide, and 2 meters deep, giving 100 cubic meters) and the flow rate of the hose (for example, 10 liters per minute, which you can measure by timing how long it takes to fill a 10-liter bucket). Then, use these estimates to calculate the filling time (100,000 liters divided by 10 liters/minute equals 10,000 minutes, or about 6.94 days).

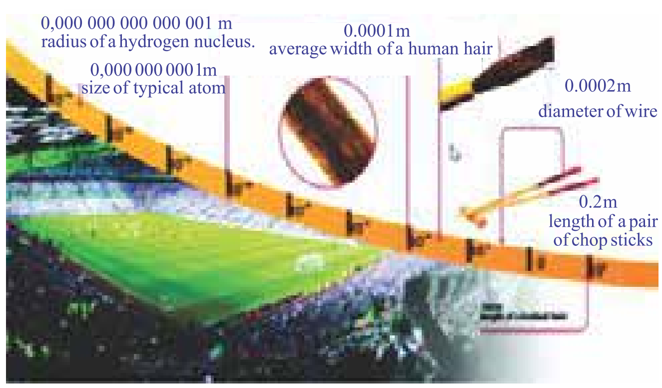
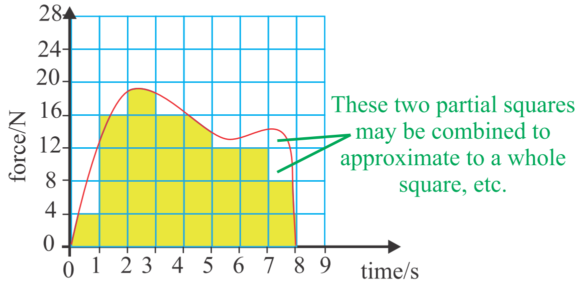


Fig 1.2: Illustration of Estimation of Sizes from Atomic level to Everyday Objects

Example 1.1

The figure illustrates the method of estimating the area under a non-uniform force-time curve to determine impulse.



Solution

Highlighted are approximately 26 full yellow squares and additional partial squares under the curve, which collectively approximate 4 full squares. With each square representing an impulse of 4 Ns, the estimated total impulse is about 120 Ns, with an expected examination tolerance, for instance, of ± 4 Ns.

Assignment

Estimate the Height of a Building Without Direct Measurement

Knowledge 1.3 | Dimensional Analysis

Based on understanding of derived and base units, we now explore their applications in dimensional analysis. This technique is essential for verifying the correctness of physical equations and deriving new formulae. Before examining how dimensional analysis ensures the correctness of physical laws, let us define the dimensions of a physical quantity.

Dimensions of Physical Quantities

Dimensionality refers to the common characteristic of a class of physical quantities that are named differently for their recognition. It represents the qualitative nature of a physical quantity. For example, quantities like the radius of the Earth, the height of a mountain, the distance between the Moon and Earth, the width of a finger, and the thickness of a paper sheet all belong to the same class of physical quantities, which are measured in units of length. Hence, by nature, all are lengths, and so they all possess the dimension of length. Dimensions in physics are the fundamental descriptors that define the nature of a physical quantity in terms of its basic components. While these components include the seven fundamental quantities, dimensions are often described in terms of length, mass, and time. These are represented by specific symbols enclosed in square brackets, such as [L], [M], and [T], respectively. The dimensions of other physical quantities can be expressed in terms of these dimensions. **For example**, the dimension of area A and volume V can written as below,

Dimension of Area

= Dimension of length x Dimension of width

$$[A] = [L] \times [L]$$

$$[A] = [L^2]$$

Dimension of Volume= Dimension of length x Dimension of width x Dimension of height

$$[V] = [L] \times [L] \times [L] \quad \Rightarrow \quad [V] = [L^3]$$

Test Yourself

1. Estimate the amount of water in liters that would fill a standard bathtub and clarify your reasoning.
2. Express the unit of heat and weight of an object, in terms of the SI base units.

Multiple Choice Questions

1. Which of the following is the best estimate for the average height of a classroom?
   a. 2 meters b. 5 meters
   c. 10 meters d. 20 meters
2. The unit of pressure, pascal (Pa), is defined as force per unit area. How is it expressed in terms of SI base units?
   a. $kg\,m^{-2}$ b. $kg\,m\,s^{-2}$
   c. $kg\,m^{-1}\,s^{-2}$ d. $N\,m^{-2}$

Skill 1.2

Ability to estimate physical quantities like length, mass, time, and temperature, applying these estimations effectively in scenarios that do not require precise measurements, which helps in developing a practical grasp of physics.

Dimension of speed = $\frac{\text{dimension of displacement}}{\text{dimension of time}}$

$$= \frac{[d]}{[t]} = \frac{[L]}{[T]} = [LT^{-1}]$$

The dimensions of a physical quantity can be obtained from its formula by analyzing the dimensions of the fundamental quantities (such as mass, length, time, etc.) included in that formula. This process, known as dimensional analysis, involves substituting the dimensions of these fundamental quantities into the equation representing the physical quantity. Table 1.4 shows the dimensions of some commonly used derived physical quantities.

Do you Know?

The physical quantities which are ratios of physical quantities having same dimensions are dimensionless like slope, strain, and resolving power etc.

Table 1.4: Dimensions of some physical quantities

Physical Quantity	Formula	Dimensions
Acceleration	Acceleration $= \frac{\text{Change in velocity}}{\text{Time taken}}$	$[LT^{-2}]$
Force	Force = Mass × Acceleration	$[MLT^{-2}]$
Work	Work = Force × Distance	$[ML^{2}T^{-2}]$
Power	Power $= \frac{\text{work}}{\text{time}}$	$[ML^{2}T^{-3}]$
Momentum	Momentum = Mass × Velocity	$[MLT^{-1}]$
Density	Density $= \frac{\text{mass}}{\text{volume}}$	$[ML^{-3}]$
Frequency	Frequency $= \frac{1}{\text{time period}}$	$[T^{-1}]$
Pressure	Pressure = Force × Area	$[ML^{-1}T^{-2}]$

The dimensional analysis is readily used to check the homogeneity of physical equations and derivation of formulae in simple cases. Both the procedures are discussed in detail as below,

1. Checking Homogeneity of physcial equation:

The correctness of any physical equation can be tested by the principle of homogeneity of dimensions. This principle states that physical equation is valid only if the dimensions on both sides are same. It is further explained by checking validity of physical equation

$v = \sqrt{\frac{F \times l}{m}}$ where v is the speed of a transverse wave on a stretched string of tension F, length l and mass m.

Dimension of L.H.S $= [v] = [LT^{-1}]$

Dimension of R.H.S $= ([F][l][m^{-1}])^{1/2}$

$= ([MLT^{-2}][L][M^{-1}])^{1/2} \ ([L^{2}T^{2}])^{1/2} = [LT^{-1}]$

As the dimensions of both sides of the physical equation $v = \sqrt{\frac{F \times l}{m}}$ are same, so the equation is dimensionally correct.

2. Derivation of Formula Using Dimensions

The dimensions can be used to derive a possible formula for particular physical quantity by deducing properly the dependency of that physical quantity on other factors. This is further explained by deriving relation for centripetal force F_c. As the centripetal force moves the object of mass "m" with velocity "v" in a circle of radius "r". It means that centripetal force will somehow depend on mass, velocity and radius i.e.

$$F \propto m^a v^b r^c$$

$$F = k\, m^a\, v^b\, r^c \text{ ----(1.1)}$$

Using dimensional analysis, we now put the dimension of m, r, and $v = \frac{s}{t}$

$$[F] = k[M]^a\left[LT^{-1}\right]^b[L]^c$$

$$\Rightarrow [F] = k[M]^a[L]^{b+c}[T]^{-b} \quad ----(1.2)$$

As we know that the dimension of force is:

$$[F] = [M]^1[L]^1[T]^{-2} \quad ----(1.3)$$

Equating the powers of $[M],[L]$ and $[T]$ in equation (1.2) and (1.3), we get; a=1, b+c=1, b=2

Hence a=1, b=2, and $c = -1$

Put the values of a, b, c in equation (1.1), we get;

$$F = k\, m^1\, v^2\, r^{-1}.$$

The numerical value of the proportionality constant cannot be determined by dimensional analysis, but it can be measured experimentally. Therefore, the approximate relation for centripetal force is:

$$F = k\,\frac{mv^2}{r}$$

Limitations of Dimensional Analysis

Besides its advantages, the dimensional analysis is subjected to the following limitations:

- It gives no information regarding the value of constant of proportionality.
- It cannot be used to derive an expression which involves trigonometric or exponential functions.
- Although an equation may be dimensionally correct, this does not ensure its physical validity. However, an equation that fails dimensional analysis, is always incorrect.
- Dimensional analysis cannot differentiate between physical quantities that have the same dimensions. For instance, if the dimension of a physical quantity is [L], it could represent the width of a room, the radius of a sphere, or the height of an object.

Test Yourself

1. Use dimensional analysis to check if the equation P = ρgh (where P is pressure, ρ is density, g is acceleration due to gravity, and h is height) is dimensionally homogeneous.
2. Derive the relation E=mc2 using dimensional analysis.
3. How dimensional analysis can be used to verify the correctness of the equation for gravitational potential energy, U = mgh (where m is mass, g is acceleration due to gravity, and h is height).

Multiple Choice Questions

1. What are the dimensions of torque?
   a) ML^2T^{-2}
   b) MLT^{-2}
   c) MLT^{-1}
   d) $ML^{-1}T^{-2}$
2. The equation F = ma is dimensionally homogeneous because:
   a. Mass and acceleration are of the same dimensions.
   b. Force and mass are of the same dimensions.
   c. The dimensions of the left side (Force) match the dimensions of the right side (mass x acceleration).
   d. Acceleration and force are of the same dimensions
3. Which of the following is a dimensionally correct formula for kinetic energy (KE)?
   a. K.E = 1/2mv² (where m = mass, v = velocity)
   b. K.E = mv (where m = mass, v = velocity)
   c. K.E = m/v (where m = mass, v = velocity)
   d. K.E = mv² (where m = mass, v = velocity)

Skill 1.3

- Competence in using dimensional analysis to analyze equation consistency and derive formulas, deepening understanding and application of physics principles.

Knowledge 1.4 | Uncertainties in Measurement:

Physics involves the measurement of numerous physical quantities, which require standard procedures and instruments for accurate readings during experiments. However, errors are inevitable in any measurement, and they may arise from limitations of the instruments, human error, environmental conditions, or faulty instruments. No measurement is entirely free from error. The error (E) is defined as the difference between the exact value and the measured value of a physical quantity. While errors can be minimized, they cannot be completely eliminated from the final result. Based on above mentioned causes, errors are classified into two main types: random errors and systematic errors, as you have studied in grade 9.

The unavoidable presence of error introduces a marginal doubt in the final result, known as uncertainty. In simple words, uncertainty is the estimate of possible range of error. Therefore, one can say that error is the cause, and uncertainty is its effect.

A convenient way to express the measurement of any physical quantity is by including the uncertainty, denoted with a positive and negative sign in front of the measured value. For instance, if the time period T of a simple pendulum is measured as 3.54 s with an uncertainty of 0.04 s, it can be expressed as:

$$T = 3.54 \pm 0.04 \text{ s}$$

Uncertainty can be classified into two types.

Absolute Uncertainty: Uncertainty in a measurement may arise from various sources, but the most primary reason is the limitation of the measuring instrument, which is directly related to the least count of the instrument. It is usually estimated by the statistical methods. Let us understand it in detail. For example, when using a meter ruler with least count of 0.1 cm, there is inherent uncertainty in determining the exact position of an object's end. If the end lies between 5.3 cm and 5.4 cm, the reading is taken as 5.3 cm if it is before the midpoint, or as 5.4 cm if it is beyond the midpoint. This introduces an error equal to half of the smallest division, i.e., 0.05 cm. Similarly, if the end of the object is beyond the 5.4 cm mark but before the midpoint of the division between 5.4 cm and 5.5 cm, the reading is still recorded as 5.4 cm. Therefore, any position from the midpoint of the division from 5.3 cm to 5.4 cm, up to the midpoint of the division from 5.4 cm to 5.5 cm, would be considered as 5.4 cm. so maximum possible error in the measurement due limitation of the instrument will be ± 0.5 cm This leads to range of maximum possible error of 0.1 cm which is equal to the least count of the ruler. We may refer it as uncertainty. To account for all other factors contributing to uncertainty, such as environmental conditions and human error, we express it as ± 0.1 cm. If the reading is taken as 5.3 cm with an uncertainty of ± 0.1 cm, it means the true measurement could lie within the range of 5.2 cm to 5.4 cm. This broader range reflects all potential sources of error contributing to the overall uncertainty. Such type of uncertainty is called absolute uncertainty. We take it equal to least count of measuring instrument written with sign of ±. For example, the length "L" of the 1st year Physics textbook, measured with a meter rod, is 25.5 cm. In case of meter rod, the least count is 0.1 cm, therefore the absolute uncertainty = ± 0.1cm The length can be expressed as 25.5 ± 0.1cm

Do you Know?

Vernier caliper and screw gauge is used to measure the length that is less than 1 mm. The least count of the vernier caliper and screw gauge is 0.1 mm and 0.01 mm respectively.

Important Information

Resolution or least count refers to the smallest value that can be measured accurately by an instrument. It signifies the precision of the instrument and dictates the minimum value of a physical quantity that can be reliably determined. For instance, consider the commonly used meter rod and tape measure with a resolution of 1 mm. This means that any length measurement made using these tools cannot be accurate to a value smaller than 1 mm.

Representation of least count of length measuring tape

Relative or Fractional Uncertainty: The relative or fractional uncertainty tells us about the correctness of the measurement". It tells us about the correctness of the measurement". If "x" is a measured value and Δx is the uncertainty in the measurement, then fractional uncertainty is $\Delta x / x$.

And percentage uncertainty = $\Delta x / x \times 100\%$

Considering the above example of measurement of length of Physics text book, the fractional and percentage uncertainties are given as:

Fractional uncertainty = $\frac{0.1}{25.5} = 0.004$

and percentage uncertainty = $\frac{0.1}{25.5} \times 100 = 0.4\%$

Thus, the final result can be expressed in a more convenient way as:

L = 25.5 with percentage uncertainty of 0.4% cm

Do you Know?

The uncertainty in the density measurement of Nitrogen resulted in discovery of Argon by Lord Rayleigh in 1894. He was awarded the Noble prize in Physics in 1904 for this discovery.

Quick Quiz

If a measurement is noted as 20.0 ± 0.5 cm, what does ± 0.5 cm represent?

Key Facts

Errors are inherent in all measurements and cannot be completely eliminated, but only minimized.

Assessment of uncertainty in a derived quantity

The physical quantities can be added, subtracted, multiplied, and divided. The other mathematical operations can also be performed on the physical quantities. Generally, a derived quantity is obtained due to the execution of mathematical operations with the base quantities. For example, area (derived quantity) of a rectangular object is product of its length and width (base quantity), so if there is uncertainty in measurement of length and width, then there would be definitely uncertainty in the area measurement.

For the assessment of uncertainty in the final result, we use the following rules:

A. Addition or Subtraction Rule

"If two measured quantities are added or subtracted then absolute uncertainty in the final result will be equal to the sum of absolute uncertainties of the measurements". Two quantities a and b are measured and we need to add or subtract them to obtain the quantity "Q" i.e. $Q = a \pm b$

Then

Total absolute uncertainty in Q = absolute uncertainty in 'a' + absolute uncertainty in 'b'.

$\Delta Q = \Delta a + \Delta b$

If $a = (14.6 \pm 0.1)$ cm and $b = (4.5 \pm 0.1)$ cm

$Q = a - b = (14.6 - 4.5)$ cm $\pm$ $(0.1 + 0.1)$ cm

$= (10.1 \pm 0.2)$ cm

B. Product and quotient rule

"If two (or more) measured quantities are multiplied or divided then relative or percentage uncertainty in the final result will be equal to the sum of their relative or percentage uncertainties".

If the two quantities 'a' and 'b' are multiplied i.e.

$Q = a \times b$, then

Total relative uncertainty in Q = relative uncertainty in a + relative uncertainty in b i.e.,

$$\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

In percentage uncertainty:

$$\frac{\Delta Q}{Q} \times 100 = \frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100$$

Example 1.2

A body of mass (2475 ± 5) kg has a volume of (2.7 ± 0.2) m^3. What is the density of the body? Also guess the material of the body and its state?

Solution: As density "ρ" of a body is given by

$$\rho = \frac{m}{V} = \frac{2475}{2.7} = 916.66 \text{ kg m}^{3}$$

Here

$m = 2475$ kg $\quad\quad \Delta m = 5$ kg

$V = 2.7\ m^3$ $\quad\quad \Delta V = 0.2\ m^3$

As in division, the final result contain sum of the relative uncertainty of the component factors, therefore, the relative uncertainty in mass and volume are:

$$\frac{\Delta m}{m} = \frac{5}{2475} = 0.002 \quad\quad \frac{\Delta v}{v} = \frac{0.2}{2.7} = 0.07 \text{ or } 7\%$$

The relative uncertainty in density will be,

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} = 0.002 + 0.07 = 0.072 \quad \text{or } 7.2\%$$

Thus $\Delta\rho = 0.072 \times \rho = 0.072 \times 916.66 = 66\, kg\, m^{-3}$

After applying rounding off rules, the density of the material can be expressed as $917 \pm 66\ \text{kg m}^{-3}$.

We know that the density of water is approximately 1000 kg m^{-3} and the calculated density is near to it (slightly less than), therefore the material of the body will be water. As the ice float at the surface of water, which means its density will be slightly less than water, so the body seem to be ice made.

C. Power of a Quantity

"If a measured quantity is raised to a power then the uncertainty in the final result is obtained by multiplying the relative uncertainty by that power."

If 'a' is raised to a certain power 'n' i.e. $Q = a^n$, then

Total relative uncertainty in Q = n times relative uncertainty in 'a' i.e., $\frac{\Delta Q}{Q} = n \times \frac{\Delta a}{a}$

In percentage uncertainty: $\frac{\Delta Q}{Q} \times 100 = n \times \frac{\Delta a}{a} \times 100$

Example 1.3

A cylinder has a radius of 4.50 cm and a length of 15.0 cm. The radius is measured using a vernier caliper with a least count of 0.01 cm, and the length is measured using a ruler with a least count of 1 mm = 0.1 cm. Determine the uncertainty in the volume of the cylinder.

Solution

We find the volume of a cylinder using its radius and length, with measurements obtained from different measuring instruments having different precision.

Radius r of the cylinder is measured using a vernier caliper with a least count of 0.1 mm (0.01 cm).

So, $r = (4.50 \pm 0.01)$ cm.

Length L of the cylinder is measured using a ruler with a least count of 1 mm (0.1 cm).

So, $L = (15.0 \pm 0.1)$ cm.

To find the uncertainty in volume of cylinder, we proceed as:

Step 1: Calculate Fractional Uncertainties of radius and length as

The absolute uncertainty in r is $\Delta r = \pm 0.01$ cm

The absolute uncertainty in L is $\Delta L = \pm 0.1$ cm

Fractional uncertainty in $r = \frac{\Delta r}{r} = \frac{0.01\text{ cm}}{4.50\text{ cm}} \approx 0.0022$

Fractional uncertainty in $L = \frac{\Delta L}{L} = \frac{0.1\text{ cm}}{15.0\text{ cm}} \approx 0.0067$

The radius appears squared in the volume formula, so the fractional uncertainty

$= 2 \times (\text{fractional uncertainty in radius}) = 2 \times \frac{\Delta r}{r}$

So, fractional uncertainty in $r^2 = 2 \times 0.0022 = 0.0044$

Step 2: Find fractional uncertainty in volume using above data which is given by,

Fractional uncertainty in the volume $V = \frac{\Delta V}{V}$

$= 0.0044 + 0.0067 = 0.0111$

Step 3: Calculate the Volume of Cylinder

Using the values for radius and length to calculate the volume:

$V = \pi \times (4.50\text{ cm})^2 \times 15.0\text{ cm} = 955.32\text{ cm}^3$

Step 6: Compute absolute uncertainty in Volume

Absolute Uncertainty in V = fractional uncertainty in V × Volume of cylinder. So,

$\Delta V = 0.0111 \times 955.32\text{ cm}^3 \approx 10.60\text{ cm}^3$

After applying rounding off rules, the volume of the cylinder can be expressed as

$V = 955 \pm 11\text{cm}^3$

Do you Know?

When finding uncertainty in a result, it's crucial to consider the uncertainties in initial measurements and how they propagate through your calculations. This involves adding absolute uncertainties for addition or subtraction and combining relative uncertainties for multiplication or division. The final uncertainty should be reported with the result, appropriately rounded to reflect the least precise measurement used in your calculation.

D. Uncertainty in Average Value

To estimate the uncertainty in the average of multiple measurements, we proceed as:

- Calculate the average value: Sum all the measured values and divide by the number of measurements.
- Calculate the deviation for each measurement: Subtract the average from each measured value. The deviation is taken as the absolute value by ignoring the sign.
- Determine the mean deviation: This is the average of all the absolute deviations and represents the uncertainty in the average measurement.

For instance, consider measurements taken with a vernier caliper to determine the thickness of a sheet of metal, with the following readings in millimeters:

0.35, 0.37, 0.36, 0.38, 0.35 and 0.36

Average value = (0.35 + 0.37 + 0.36 + 0.38 + 0.35 + 0.36) / 6 = 0.36 mm

Deviations from the average are:

$|10.36 - 0.35| = 0.01$, $|10.36 - 0.37| = 0.01$,

$|0.36 - 0.36| = 0$, $|10.36 - 0.38| = 0.02$,

$|10.36 - 0.35| = 0.01$, $|10.36 - 0.36| = 0$.

$$\text{Mean deviation} = \frac{0.01 + 0.01 + 0 + 0.02 + 0.01 + 0}{6}$$

$$= 0.0083 \text{ mm}$$

Writing in one significant digit will make it 0.01mm.

Therefore, the estimated uncertainty in the average thickness of 0.36 mm is about 0.01 mm, which can be recorded as 0.36 ± 0.01 mm. This represents how much the actual value might vary from the measured average.

E. Uncertainty in Timing Experiment

Do you Know?

Precision refers to the consistency of repeated measurements, indicating how close the measurements are to each other, while accuracy describes how close a measurement is to the true or accepted value. High precision doesn't guarantee high accuracy, and vice versa; achieving both in measurements is essential for reliable results.

In determining the uncertainty in a timing experiment for the period of a vibrating body, the least count of the timing device is divided by the number of vibrations i.e.,

$$\text{Uncertainty in time period} = \frac{\text{L.C of stopwatch}}{\text{no. of vibrations}}$$

For instance, if a stopwatch accurate to one hundredth of a second is used to record the time for 25 vibrations of a pendulum, and the recorded time is 37.5 seconds, then the period T is calculated as:

$$T = \frac{37.5}{25}\text{ s} = 1.50 \text{ s}$$

with an uncertainty of $\frac{0.01 \text{ s}}{25} = 0.0004$ s.

This uncertainty indicates that the time period should be recorded to three decimal places. Therefore, it should be expressed as:

$$T = 1.500 \text{ s} \pm 0.004 \text{ s}$$

This illustrates the importance of counting a large number of swings to reduce timing uncertainty.

Why All Measurements Contain Some Uncertainty

All the measurements are uncertain to some extent. Reasons for uncertainty in measurements are :

1. Limitation of Instruments:

Every measuring instrument has a finite precision. For example, a standard ruler might not measure smaller than a millimeter, introducing a small margin of error in any measurement.

2. Human Error:

The person taking the measurement can introduce errors, such as misreading a scale or making inconsistent judgments, which is particularly common in manual measurements.

3. Environmental Influences:

External factors like temperature, humidity, or air pressure can affect the object being measured or the measuring instrument itself. For instance, metal expands slightly when heated, which can alter precise measurements.

4. Subjective Interpretation:

Some measurements require a degree of interpretation, such as estimating the exact level of a liquid in a graduated cylinder, which can vary slightly between different observers.

Skill 1.4

- Understanding that all measurements have inherent uncertainties, fostering a realistic approach to experimental physics and proficiency in calculating uncertainties in derived quantities, essential for accurate scientific reporting.

Knowledge 1.5 | Analyzing and Critiquing the Accuracy and Precision

In science, the terms accuracy and precision describe distinct aspects of measurement. Accuracy refers to how close a measurement is to the true or standard value, indicating the correctness of a measurement. Precision denotes the consistency or repeatability of measurement outcomes and is a measure of how close repeated measurements are to each other. Understanding these concepts is essential for assessing the reliability of scientific data.

Analyzing Accuracy and Precision

To analyze accuracy and precision, one must examine how measuring instruments perform under various conditions. Consider an example involving two groups of students measuring gravitational acceleration using simple pendulums. The first group's measurements in ms^{-2} are 9.12, 9.13, 9.13, 9.14, and 9.14. These measurements are precise because they are closely grouped but not accurate as they deviate from the accepted value of 9.8 m s^{-2}. Conversely, the second group's measurements of 9.81, 9.70, 9.90, 9.78, and 9.75 ms^{-2} are more varied but closer to the true value, making them less precise but more accurate.

The Fig.1.3 visually and graphically explains the concepts of accuracy and precision using archery targets

The precision of a measurement is determined by the measuring instrument. To examine precision, we measure absolute uncertainty, which we usually take as equal to the least count of the measuring instrument. Absolute uncertainty provides a measure of the spread of repeated measurements. A smaller absolute uncertainty indicates that the measurements are closely clustered together, reflecting high precision. Let us consider two measurements: one taken with a meter ruler and another with vernier calipers. The measurement of the length of an object with a meter ruler is 20.5 cm ± 0.1 cm, while the measurement of the diameter of a small cylinder with vernier calipers is 2.50 cm ± 0.01 cm. The vernier caliper has a smaller absolute uncertainty (± 0.01 cm) compared to the meter ruler (±0.1 cm). Therefore, the measurement taken with the vernier caliper is more precise. Lesser

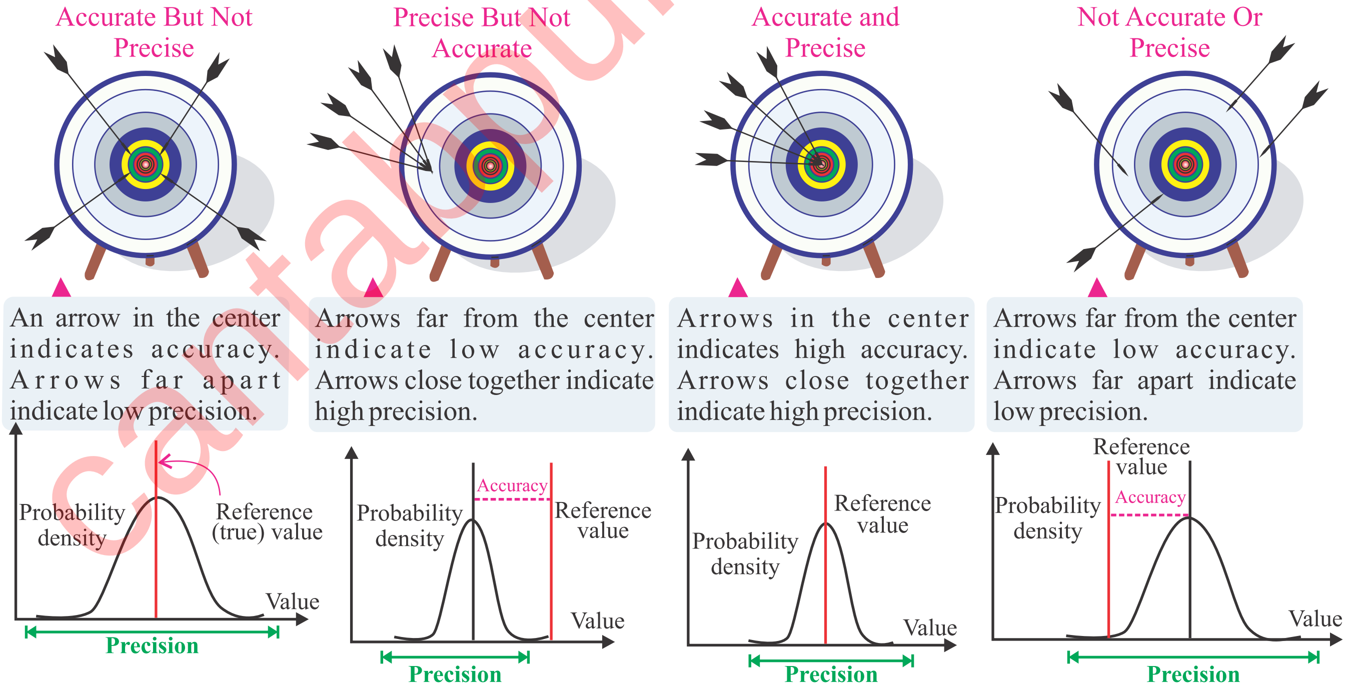


Figure 1.3 provides a graphical interpretation of precision and accuracy using probability density graphs. The peak of each graph, representing the average or most frequent measurement value, indicates where most data points cluster. The width of each graph shows the spread of measurements, with a narrower width indicating higher precision. The vertical red line represents the reference or true value. These graphs visually represent precision and accuracy.

the absolute uncertainty, more precise the measurement, more close measurement will be taken from such intrument.

Accuracy of a measurement is assessed by comparing the measured value to a known or accepted true value. To evaluate accuracy, we consider both absolute and percentage uncertainty. Absolute uncertainty provides information about the possible range of values within which the true value lies, indirectly contributing to understanding accuracy. Fractional or percentage uncertainty helps in evaluating accuracy by showing how significant the uncertainty is relative to the measured value. An accurate measurement is one which has less fractional or percentage uncertainty. To check the accuracy of measurement, let us measure the fractional or percentage uncertainty of the above two measurements taken from the meter ruler and vernier calipers.

Fractional uncertainty in the measurement taken from meter ruler $= \dfrac{0.1 \text{ cm}}{20.5 \text{ cm}} = 0.0049$

And its percentage uncertainty

$$\left(\frac{0.1 \text{ cm}}{20.5 \text{ cm}}\right) \times 100\% = 0.5\%.$$

For the vernier caliper,

fractional uncertainty $= \dfrac{0.01 \text{ cm}}{1.10 \text{ cm}} = 0.0090$

And the percentage uncertainty

$$= \left(\frac{0.01 \text{ cm}}{1.10 \text{ cm}}\right) \times 100\% = 0.90\%.$$

The measurement with the meter ruler has a smaller percentage uncertainty (0.5%) compared to the vernier calipers (0.9%), indicating higher accuracy.

Do you Know?

Random errors cause fluctuations in measurement results, reducing the precision, Systematic errors, on the other hand, lead to a consistent deviation in one direction, thereby affecting the accuracy.

Impact of Errors on Precision and Accuracy

Measurements are subject to two primary types of errors: random and systematic. Random errors result in data points scattering, which affects the precision of measurements. These errors are unpredictable and can cause significant variations, reducing precision if they are substantial. Precision is specifically determined by absolute uncertainty, which quantifies the range of variability in measurements.

On the other hand, systematic errors influence accuracy by causing a consistent deviation from the true value. These errors do not decrease even with an increased number of measurements because they are inherent to the equipment or the experimental setup.

Critiquing the Accuracy and Precision

Precision is directly related to the capabilities of a measuring instrument, particularly its least count. An instrument with a smaller least count offers greater precision, enabling more detailed and consistent measurements. Furthermore, proper calibration of an instrument and removing zero error in it, ensures more accurate readings.

When critiquing measurement tools, it is essential to assess their suitability for specific tasks, considering both precision and accuracy requirements. The choice of the appropriate instrument depends on the level of detail needed for the measurement and how closely the measurements need to align with the true or standard values.

For basic tasks like measuring the dimensions of a notebook, a meter rod with a millimeter scale is used. Ensuring this meter rod is properly calibrated improves its accuracy, providing measurements that agree with the actual dimensions.

For more precise measurements, such as determining the thickness of paper, a screw gauge with a 0.01 mm least count is ideal. This finer scale enhances the precision of the measurement, allowing for more detailed readings.

In critiquing measurement tools, both the precision and the accuracy must be considered to ensure reliable and valid measurement outcomes.

The suitability of some instrument and their limitations are given table 1.5.

Table 1.5: Limitation of measuring instruments

Instrument	Length to be measured	Smallest unit
Measurement Tape	Several meters	1 cm or Imm

Meter rule or half-meter rule	Several centimetres to 1 meter	0.1 cm or 1mm
Vernier Calliper	millimetres to centimetres	0.01 cm or 0.1 mm
Micrometer Screw Gauge	Less than 1 millimeter to about 2 centimeters	0.001 cm or 0.01 mm

Test Yourself

1. In a classroom experiment, you are using a meter stick to measure the height of a table. What steps would you take to ensure the meter stick provides accurate measurements, and why is this process important?
2. For a school project, you are tasked with measuring the diameter of various thin wires. Which instrument would you choose for this precise measurement and why is its least count significant?

Skill 1.5

- Understanding that all measurements have inherent uncertainties, fostering a realistic approach to experimental physics and proficiency in calculating uncertainties in derived quantities, essential for accurate scientific reporting.

Multiple Choice Questions

1. When a scale consistently gives a reading of 101 kg for a 100 kg weight, the scale is:
   a. Accurate but not precise
   b. Precise but not accurate
   c. Both accurate and precise
   d. Neither accurate nor precise
2. If the length of a table is measured as 2.00 m $\pm$ 0.05 m and the width as 1.00 m $\pm$ 0.02 m, what is the uncertainty in the area of the table?
   a. $\pm$ 0.07 m^2 b. $\pm$ 0.10 m^2
   c. $\pm$ 0.14 m^2 d. $\pm$ 0.02 m^2
3. If a measurement of time is 20 seconds with an uncertainty of 1 second, what is the fractional uncertainty?
   a. 0.05 b. 0.1
   c. 0.2 d. 0.02

Key Points

- Estimation of Physical Quantities involves making educated guesses about physical quantities (like distances, times, or masses) based on observations and general knowledge. It is useful when precise measurements aren't possible or necessary.
- Expression of Derived Units is about understanding and using complex physical units, like Newtons (force), Joules (energy), or Watts (power). These units are derived from the basic SI (International System of Units) units, like meters, kilograms, and seconds, often through multiplication or division.
- Homogeneity of Physical Equations refers to ensuring that the units in a physical equation are consistent on both sides. It involves dimensional analysis, a method where you check that the units match up, ensuring the equation makes physical sense.
- Dimensional analysis is a technique used to derive or check the consistency of physical formulas by analysing the dimensions (units) of the physical quantities involved.
- Accuracy is the degree to which a measurement aligns with the true or actual value. High accuracy means the measurement is very close to the real value.
- Precision is the degree to which repeated measurements under unchanged conditions show the same results. High precision means very little variation between these measurements.
- Assessment of measurement uncertainty involves determining the degree of doubt in a measurement. It includes calculating the absolute uncertainty (the margin of error in units), fractional uncertainty (the ratio of the absolute uncertainty to the measured value), and percentage uncertainty (the fractional uncertainty expressed as a percentage).
- Uncertainty in measurements refers to the doubt or variability inherent in any measurement process. It represents the range within which the true value is expected to lie and arises due to limitations in measuring instruments, human error, and environmental factors.
- All measurements contain some level of uncertainty because no measuring instrument is perfectly precise, and various factors, such as instrument limitations and environmental conditions, introduce variability in the measurement process.

Exercise

A Multiple Choice Questions:

Select the best answer of the following questions.

1. When estimating the height of an object using a reference point, which of the following is the most important factor to consider?
a. The color of the object
b. The weight of the object
c. The distance to the object
d. The similarity in type between the object and the reference

2. Which technique is essential when estimating the volume of a complex-shaped container filled with liquid?
a. Calculating the container's weight
b. Dividing the container into smaller, regular shapes
c. Estimating the container's color intensity
d. Measuring the temperature of the liquid

3. Which of the following have the same dimensions.
a. angle   b. pressure   c. stress   d. both b and c

4. Which one of the following is dimensionless quantity
a. stress   b. strain   c. co-efficient of viscosity   d. energy

5. 5 joule of kinetic energy will be equal to
a. 5×10^5 dyne   b. 5×10^5 erg   c. 5×10^7 dyne   d. 5×10^7 erg

6. The dimensional analysis fails for deriving expression, which involves
a. One quantity   b. two quantities   c. three quantities   d. trigonometric functions

7. One million nanosecond equals:
a. one femtosecond   b. one thousand milliseconds
c. one thousand microseconds   d. one millisecond

8. The dimensions of force is:
a. $[M^0L^1T^{-2}]$ b. $[M^1L^1T^{-2}K^0]$ c. $[M^0L^1T^{-2}K^1]$ d. $[M^1L^1T^{-1}]$

9. The precision of a certain measurement is determined from:
a. direct error   b. relative error.   c. percentage error   d. least count

10. Which error is suitable to compare accuracy of different measurements:
a. direct error   b. relative error   c. percentage error   d. both b and c

11. If the percentage error in mass is 1% and velocity is 2%, then the possible percentage error in kinetic energy will be:
a. 3%   b. 4%   c. 5%   d. 6%

12. If $x = \frac{a}{b}$ and Δa and Δb represent the errors in the measurements of a and b respectively, then what is the maximum percentage error in the value of x?
a. $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100$ b. $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right) \times 100$
c. $\left(\frac{\Delta a}{a-b} + \frac{\Delta b}{a-b}\right) \times 100$ d. $\left(\frac{\Delta a}{a-b} - \frac{\Delta b}{a-b}\right) \times 100$

13. The light-year is a unit used to measure which of the following?
a. distance   b. time   c. linear speed   d. angular speed

14. Which of the following options represents the measurement of one light-year?

a. 3.08×10^3 km   b. 9.5×10^{12} km

c. 1.057×10^8 km   d. 1.496×10^7 km

15. Among the following choices, which one does not represent a unit of time?

a. hour   b. nano-second

c. micro-second   d. light year.

16. If the error in the measurement of the radius of a sphere is 2%, what is the approximate error in the measurement of its volume?

a. 1%   b. 3%   c. 5%   d. 6%

17. Planck's constant has the same unit as that of the following:

a. torque   b. impulse

c. power   d. angular momentum

B Assertion – Reason Type Questions

Make 14 assertion-reason type questions from a given pattern in each of the following questions. Two statements are labeled as assertion (A) and other labeled as a reason in (R) are shown. Examine both the statements and mark the correct choice according to the instructions given below (a) if both A & R are correct & R is the reason for A (b) if both A & R are correct, but R is not the reason for A (c) If A is Correct and R is wrong (d) if A is wrong and R is correct.

1. **A :** The SI system includes seven base units like meter, kilogram, and second.
   R : These units are sufficient to describe all physical phenomena in natural science.
2. **A :** In dimensional analysis, the dimensions on both sides of a physical equation must be the same.
   R: This is to ensure the equation is physically meaningful.
3. **A :** The least count of an instrument affects its precision.
   R : The least count determines the smallest measurable value, influencing the repeatability of measurements.
4. **A:** Random errors in measurements lead to inaccuracies in the final results.
   R: Random errors cause variability in measurement outcomes, affecting precision, not accuracy.
5. **A:** Systematic errors result from flaws in the measurement system.
   R: These errors affect the accuracy of measurements by causing consistent deviations from the true value.
6. **A:** Estimation is a key skill in physics, particularly for complex calculations.
   R:Estimations are used to simplify problems and assess the validity of results.
7. **A:** A coherent system of units allows derived units to be formed without introducing numerical factors.
   R: The SI system is an example of a coherent unit system.
8. **A:** Strain is a dimensionless quantity.
   R: Strain is calculated as the ratio of two lengths, making its units cancel out.
9. **A:** The MKS system was limited as it primarily focused on mechanics.
   R: The MKS system was expanded to the SI system to cover all areas of physics.
10. **A:** The unit of force in the SI system is Newton.
    R: A Newton is the force required to accelerate a one-kilogram mass at a rate of one meter per second squared.
11. **A:** The dimensions of force are $[M^1 L^1 T^{-2}]$.
    R: Force is defined as mass times acceleration, where acceleration has dimensions of $[L^1 T^{-2}]$.
12. **A:** Dimensional analysis cannot derive formulas involving trigonometric functions.
    R: Trigonometric functions do not have dimensions.
13. **A:** In an experiment, taking a large number of observations reduces random errors.
    R: Multiple observations help in averaging out the random fluctuations in the data.
14. **A:** Absolute uncertainty depends on the least count of the measuring instrument.
    R: The least count determines the smallest measurable increment, affecting the precision of the instrument.

C Restricted Response Question

1.1. What is meant by a unit? What are the basic essentials which a unit must possess?

1.2. What are the limitations of MKS system? How were these overcome?

1.3. What is meant by a coherent system of units? Identify a coherent system of unit?

1.4. An equation is dimensionally correct. Does it mean that the equation is necessary correct?

1.5. If a measurement $X = x + \Delta x$ and $Y = y + \Delta y$, then prove that both X + Y and X-Y will have an error of $\Delta x + \Delta y$.

1.6. Why do we take a large number of observations while performing an experiment?

1.7. How can systematic errors impact the accuracy of data collected with a measuring instrument?

1.8. Prove that the relation $E = hc/\lambda$ is dimensionally correct.

1.9. What are the dimensions of Force?

1.10. Determine the dimension of mc^2. What is the significance of this result?

1.11. How would you estimate the uncertainty in a derived quantity when adding or subtracting measurements with their respective uncertainties?

1.12. What is the criterion for selecting fundamental physical quantities for a system of units?

1.13. Identify the appropriate instruments for the measurement of length 4.0 cm and 4.000 cm.

1.14. In what three categories can all physical measurements encountered in mechanics be expressed?

1.15. How would you express the percentage error in the measurement given as 8.9 mm ± 0.2 mm?

D Extended Questions

1.1. What is the System International (SI)? Explain the reasons for the popularity of this system of units. List the SI base units, provide examples of derived units expressed as products or quotients of these base units.

1.2. Explain how dimensional analysis can be used to derive the formula for a physical quantity.

1.3. Justify the statement: "All measurements contain some uncertainty." Provide examples to support your explanation.

1.4. Explain precision and accuracy with examples and figures. Also relate precision and accuracy with different types of uncertainties?

1.5. Find the following dimension: Planck constant, gravitational constant?

1.6. Analyze the homogeneity of the following equation and determine if it is dimensionally consistent: $v^2 = u^2 + 2aS$

E Numerical Problem

Problem 1.1: Find the unit of length, mass and time if the unit of force, velocity and energy, respectively, are 100 N, 10 ms^{-2} and 500 J. **Hint:** use dimensional analysis. **[5m, 0.5s, 5kg]**

Problem 1.2: Time period "T" of a simple pendulum depends upon its length and acceleration due to gravity at that place. Obtain an expression for "T", using the method of dimensional analysis.

$$T = k\sqrt{\frac{l}{g}}$$

Problem 1.3: A biologist is filling a rectangular dish with growth culture and wishes to know the area of the dish. The length of the dish is 12.61 cm and width is 5.72 cm. Find the area of the dish?. **[72.1 cm²]**

Problem 1.4: A body of mass (27.2 ± 0.1) g has a volume of (2.0 ± 0.1) cm^3. What is the density of the body? Find uncertainty in the density measurement. **[(13.6 ± 0.7) g cm^{-3}]**

Problem 1.5: Derive a relation for the time period of simple pendulum using dimensional analysis. The time period might depend on length of pendulum I, mass of bob m, angle of thread with vertical axis θ, and gravitational constant g.

Problem 1.6: Check the correctness of the following relation by dimensional analysis.

$$T = \sqrt{\frac{\rho r^3}{F_t}}$$

T = time period of oscillation, ρ = density, r = radius, F: force of surface tension.

Problem 1.7: An aero-plane is doing bailey landing on the surface of the sea with a speed of 200 K m h^{-1}. It stopped in 5s covering a distance of 0.5 km with an acceleration of 230000 kmh^{-2}. Convert all the quantities in SI units. **[55.6 m s^{-1}, 5s, 500m, 17.7ms^{-2}]**

Problem 1.8: The micrometer (1 µm) is often called the micron.

(a) How many microns make up 1.0 km?

(b) What fraction of a centimeter equals 1.0 µm? (c) How many microns are in 1.0 yd?

Hint: 1 yd = 91.44cm

$$\left[\mathbf{10^9 \mu m, \frac{1}{10000} cm, 9.144 \times 10^5 \mu m} \right]$$

Problem 1.9: How would you express the percentage error in the measurement given as 89 mm ± 2 mm? .A mass m of 3500grams ± 100 grams is moving at a velocity v of 20 ms^{-1} ± 1 ms^{-1} in a circular path with a radius r of 1250m m ± 50mm. The force F acting on this object is calculated using the formula:

$$F = \frac{mv^2}{r}$$

Determine the following:

1. The maximum possible fractional error in the measurement of force.
2. The maximum possible percentage error in the measurement of force.
3. How the force measurement should be recorded based on the calculated error.

[0.169, 16.9%, 1120 N ± 189 N]

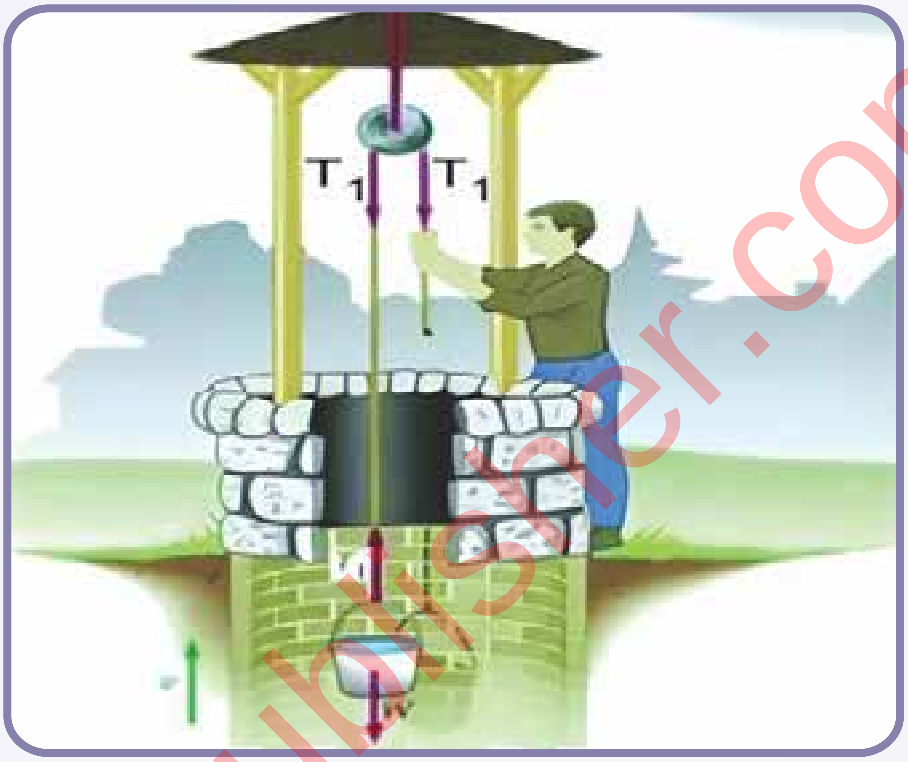
Problem 1.10: A student measures the length of a pendulum as L = 0.750 ± 0.005 meters using a ruler with a least count of 0.001 meters. The time for 30 complete oscillations is measured as T = 45.0 ± 0.2 seconds using a stopwatch with a least count of 0.1 seconds. Calculate the value of g (acceleration due to gravity). Also find the uncertainty in the value of g. **[g = 9.79 ± 0.09 m s^{-2}]**

Problem 1.11: A metal cylinder is measured to have a diameter d = 0.050 ± 0.001 meters and a height h = 0.150 ± 0.002 meters using a vernier caliper with a least count of 0.001 meters. The mass of the cylinder is measured as m = 0.500 ± 0.005 kilograms using a balance with a least count of 0.001 kilograms. Calculate the density ρ of the metal cylinder and the uncertainty in the value of ρ. **[ρ = 170 ± 180 kg m^{-3}]**

Problem 1.12: A resistor's resistance R is calculated using Ohm's Law R = V/I. A voltmeter with a least count of 0.01 volts measures the voltage across the resistor as V = 12.0 ± 0.1 volts. An ammeter with a least count of 0.01 amperes measures the current through the resistor as I = 2.0 ± 0.05 amperes. Calculate the resistance R and find the uncertainty in the value of R. **[R = 6.0 ± 0.20 Ω]**

CHAPTER

2 VECTORS



Student Learning Outcomes

Knowledge 2.1

Fundamental Concepts of Scalar and Vector Quantities

[SLO: P-11-B-01]: Differentiate between scalar and vector quantities

Knowledge 2.2

Vector Representation in terms of Components

[SLO: P-11-B-01]: Represent a vector in 2-D as two perpendicular components

Knowledge 2.3

Product of Two Vectors

[SLO: P-11-B-02]: Describe the product of two vectors (dot and cross-product) along with their properties

Introduction

In this chapter, students will focus on understanding the differences between scalar and vector quantities, which are fundamental concepts in physics. They will learn that scalar quantities have only magnitude, while vector quantities have both magnitude and direction. The chapter will also cover how to represent a vector in two dimensions as two perpendicular components, helping students visualize and analyze vectors more effectively. Additionally, students will explore the concepts of dot and cross products, which describe how two vectors can be multiplied, along with their key properties. These topics are essential for solving problems involving forces, motion, and other physical phenomena.

Knowledge 2.1 | Fundamental Concepts of Scalar and Vector Quantities

In our daily lives, we interact with various physical quantities that help us understand different aspects of our environment, such as the speed of a moving car or the force of the wind against a building. These quantities fall into two categories: scalars and vectors.

Scalars are quantities described only by their magnitude, which is a measure of size or amount expressed as a number with a unit. For example, the temperature of a room at 25°C or the mass of an object at 10 kilograms are scalar quantities. They don't involve direction and can be added, subtracted, multiplied, and divided using ordinary arithmetic rules. For instance, if you combine two masses of 5 kg and 10 kg, their total mass is simply the sum, 15 kg.

Vectors, in contrast, require both magnitude and direction for a complete description. A vector specifies not just "how much" but also "which way." For example, a car moving at 60 km/h eastward is described by both its speed (magnitude) and the direction (eastward). Similarly, a force (weight) of 10 Newtons acting downward on a book is a vector because it includes both the strength of the force and its direction.

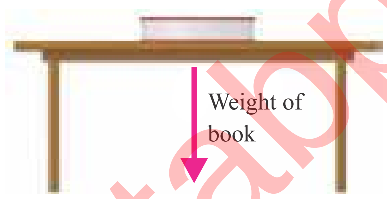


Figure 2.1: Weight of book acting downwards

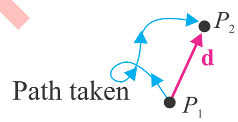
Vectors are typically represented by bold letters or with an arrow above the letter, such as **v** or $\vec{v}$. For instance, a vector used to locate a point can be denoted as **r** or $\vec{r}$. In this book, we will use bold capital letters to represent vectors. The magnitude of a vector is the modulus of the vector and is represented as r or |**r**|.

In physics, mathematics, and engineering, a vector (also known as an Euclidean vector) is a geometric entity defined by its magnitude and direction. They are represented graphically by a straight line with an arrowhead, as shown in Fig.2.1(a) for the displacement of an object from point P_1 to point P_2. The arrowhead indicates the vector's direction, while the length of the line, scaled appropriately, represents its magnitude. The starting point of the line is called the tail, and the endpoint with the arrowhead is called the head.

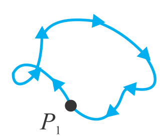
Displacement as vector quantity

(a) We represent a displacement by an arrow that points in the direction of displacement.

(b) A displacement is always a straight arrow directed from the starting position to the ending position. It does not depend on the path taken, even if the path is curved.



(c) Total displacement starting from point P_1 for a round trip is 0, regardless of the path taken or distance traveled.



Vector manipulations follow certain rules called vector algebra, which differ from the simple arithmetic rules applied to scalars. To add or subtract vectors, both their magnitudes and directions are considered using methods such as the head-to-tail method or the parallelogram rule, which students already learned in Grade 9. While vectors can also be multiplied using vector algebra, which will be discussed later in this chapter. There is no operation for dividing one vector by another. This is because vectors lack a multiplicative inverse, making division undefined in vector algebra.

To describe a vector, it is essential to specify its direction. This can be done using cardinal directions (north, south, east, west) as shown in Fig. 2.2, with north typically represented as upward on a vertical axis. Alternatively, in physics and mathematics, the Cartesian coordinate system is often used to express a vector's direction in both plane (two dimensions) and in space (three dimensions).

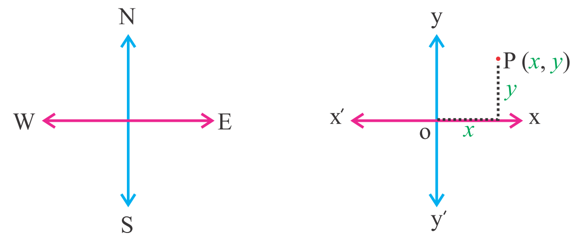


Figure 2.2: Cartesian coordinate system

Cartesian Coordinate System

The rectangular coordinate system, or Cartesian coordinate system, is defined by two reference lines drawn at right angles to each other, known as the coordinate axes. The point 'O' where these axes intersect is the origin. The horizontal line is the x-axis, with the right direction considered positive, while the vertical line is the y-axis, with the upward direction considered positive. The plane formed by these axes is the xy-plane. This is illustrated in Fig.2.2(b) To locate any point on this plane, distances are measured along the x and y axes, known as coordinates, and represented as P(x,y).

When representing a vector **A** in the rectangular coordinate system, the tip of the vector is treated as a point with coordinates(A_x, A_y) as indicated in Fig.2.3. These coordinates correspond to the projections of the vector along the x -axis and y -axis, which will be discussed later as the components of the vector.

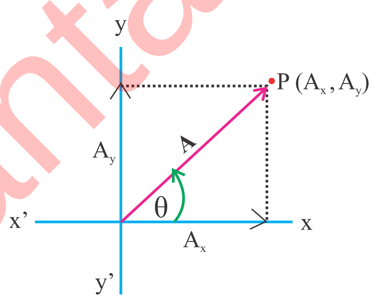


Figure 2.3: Vector representation in two dimensional Cartesian coordinates

To describe the direction of a vector in the xy-plane, we examine its orientation relative to the x-axis. Specifically, we measure the angle θ between the vector and the positive x-axis, typically in an anticlockwise direction as indicated in the Fig.2.3.

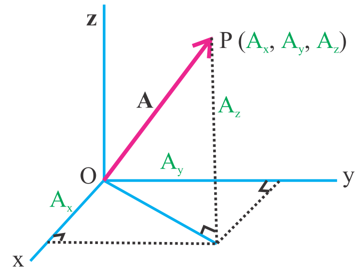


Figure 2.4: Vector representation in three dimensional Cartesian coordinates

When extending this concept to three-dimensional space, we introduce a third axis, the z-axis, which is perpendicular to both the x and y axes as shown in Fig. 2.4. In this three-dimensional space, any vector can be represented by its tip as a point with coordinates (A_x, A_y, A_z) These coordinates are the components of the vector in space. The direction of the vector in three-dimensional space is determined by the angles it forms with the positive x, y and z axes, as shown in the Fig. 2.5.

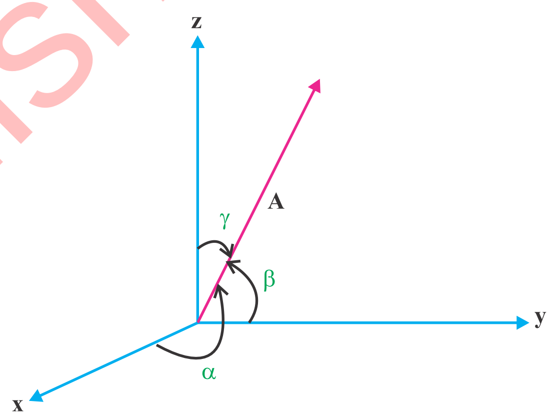


Figure 2.5: Angles made by vector A with x,y and z-axes.

Multiplication of a vector by a scalar (number)

When a vector **A** is multiplied by a scalar or number 'n', the result is still a vector quantity with magnitude n |**A**|= nA. Because of the nature of number 'n', the resultant vector will have:

- The same direction but different magnitude than **A** if n > 0 except n = 1. In this type of product, the resultant **B** is parallel to **A** as shown in the Fig 2.6.
- The opposite direction and different magnitude than **A** if n < 0 except n = - 1. In this type of product, the resultant **C** is anti-parallel to **A** . as shown in the Fig. 2.6.
- Zero magnitude and arbitrary direction if n = 0. In

this type of product, the resultant **D** (discussed later) is a null vector. It is represented by a dot in the Fig .2.6.

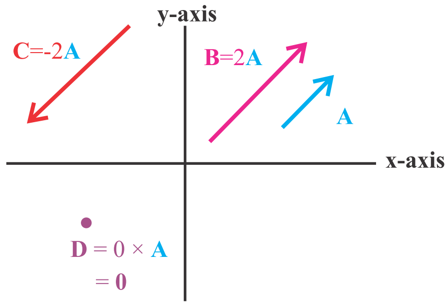


Figure 2.6: Multiplying vector with number

Negative vectors

A vector which is equal in magnitude to the original vector but acting in the opposite direction. In Fig.2.7, **D** is a negative vector of **A**. Symbolically it can be written as:

$$\mathbf{D} = -\mathbf{A}$$

Generally, every vector can have a negative vector if multiplied by -1. For example, negative vector of **A** is denoted by -**A**.

Parallel and Anti- Parallel Vectors

Two vectors are said to be parallel if they have the same direction, regardless of their magnitude or starting point. In Fig.2.7, vectors **A**, **B** and **C** are examples of parallel vectors. When two vectors have opposite directions, they are called anti- parallel vectors. The angle between anti-parallel vectors is 180°. For example, in Fig. 2.8, vector **D** is anti-parallel to vectors **A**, **B**, and **C**. Non-parallel vectors, on the other hand, can form any angle other than 0° or 180° with each other. For instance, vector **E** in Fig. 2.7 is non-parallel with respect to **A**, **B**, **C**, and **D**.

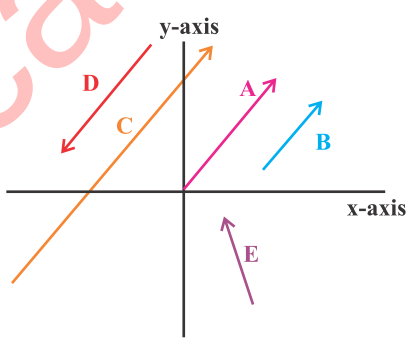


Figure 2.7: Representation of parallel and anti-parallel vectors

Equal vectors

Two vectors are said to be equal if they have the same magnitude and direction. Equal vectors have similar properties like parallel vectors but with an extra restriction that the magnitude of the two vectors will also be equal. As shown in Fig.2.8, equal vectors can be written as:

$$\mathbf{A} = \mathbf{B} = \mathbf{C}$$

Graphically, equal vectors are parallel, equal in magnitude but there is no restriction on their starting point as shown in Fig 2.8.

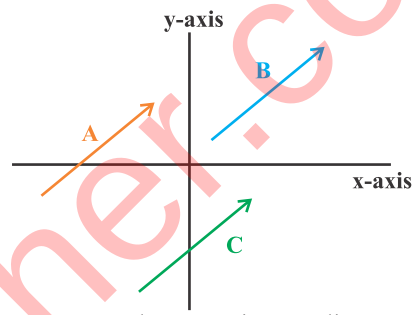


Figure 2.8: Equal vectors in coordinates system

Unit vectors

A unit vector is a vector that has a magnitude equal to one. A unit vector is used to describe the direction of a vector, therefore sometimes also known as a direction vector. A unit vector in the direction of **A** is written as $\hat{A}$ which we read as "A hat". Mathematically we can write:

$$\mathbf{A} = |\mathbf{A}|\hat{A} \qquad \hat{A} = \frac{\mathbf{A}}{|\mathbf{A}|}$$

Key Facts

- Negative vectors are like that of anti-parallel vectors with the restriction that the magnitude of the antiparallel vectors are equal.

Null vectors

A vector having magnitude equal to Zero and arbitrary direction is called null vector or zero vector. The key question is how a quantity can be considered a vector when its magnitude is zero and its direction is arbitrary. We know from vector arithmetic that addition or subtraction of two vectors always results in a vector quantity, therefore,

$$\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$$

The null vector is denoted by $\vec{0}$ or **0** and its magnitude $|\mathbf{0}| = 0$. Graphically, a null vector is represented by a point because a point can have zero magnitude as shown in Fig 2.6 where vector **D** = 0 x **A** = **0** is a null vecor represented by a point.

Assignment

Explain why a null vector has an arbitrary direction and how it differs from other vectors. Provide an example from physics where a null vector might be encountered and discuss its significance.

Multiple Choice Questions

1. Which of the following operations can be performed on vectors?
   a. Addition b. Subtraction
   c. Multiplication by a scalar d. All of the above
2. What is the negative of a vector?
   a. A vector with the same magnitude but opposite direction.
   b. A vector with the same direction but half the magnitude.
   c. A vector with double the magnitude and the same direction.
   d. A vector that is perpendicular to the original vector.
3. In a Cartesian coordinate system, how is a vector represented?
   a. Using angles and magnitudes.
   b. Using a set of coordinates (x, y, z).
   c. Using a single numerical value.
   d. Using a circular path.

Test Yourself

1. How to represent the negative of a vector in cartesian coordinates?
2. Define a position vector and distinguish it from a displacement vector.

Knowledge 2.2 | Vector Representation in terms of Components

Understanding vectors as quantities with both magnitude and direction lays the groundwork for their practical application, such as in aviation. By representing vectors in two-dimensions through perpendicular components, we can efficiently manage real-world tasks like navigating an airplane. This approach allows for precise control over the aircraft's direction and speed, illustrating the essential role of vector representation in daily activities.

Resolution of Vectors

In your previous studies, you learned how to determine the resultant of multiple vectors through vector addition. The reverse process of this is called vector resolution, where a single vector is decomposed into two or more components. These components represent the vector's influence along specific directions, usually along a set of perpendicular reference axes. The components can be understood as the projections of the vector onto these axes, which are found by dropping perpendiculars from the vector's tip to each axis. When these components are combined using the head-to-tail method, they recreate the original vector. The resolution of a vector is, therefore, dependent on the orientation of the chosen axes, and it is fundamental in analyzing the vector's behavior in different directions.

Quick Quiz

Imagine you are a civil engineer tasked with determining the necessary strength of cables for a new suspension bridge. The bridge design includes cables that are expected to hold a maximum load of 10,000 N at an 30° angle from the vertical.

Calculate the horizontal and vertical components of the force exerted by the cables. How will this information be critical in ensuring the structural integrity of the bridge?

If the components of a given vector are perpendicular to each other, they are called as rectangular components

Let us Consider a vector **A** with magnitude A, making an angle θ with the x -axis, as shown by ($\overrightarrow{OB}$) in Fig. 2.10. To resolve **A** into its rectangular components, draw a perpendicular BC to the x-axis and BD to the y-axis, as illustrated in the figure. The projection ($\overrightarrow{OC}$), representing the component of **A** along the x -axis, is denoted as $\mathbf{A}_x$ and is referred to as the x -component. Similarly, the projection ($\overrightarrow{OD}$), representing the component of **A** along the y-axis, is denoted as $\mathbf{A_y}$ and is known as the y-component of vector **A**. Since these components are perpendicular to each other, so they are called the rectangular components of **A**. By the head-to-tail rule, we can express:

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y}$$

or $\mathbf{A} = A_x\hat{i} + A_y\hat{j}$ --------------(2.1)

Equation (2.1) confirms that vector A is the resultant of its components. Given the magnitude A of vector **A** and the angle θ, the rectangular components can be determined using the trigonometric ratios from triangle OCB:

$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta \quad \text{-----------} (2.2)$$

Similarly,

$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta \quad \text{-----------} (2.3)$$

Equations (2.2) and (2.3) show that if the vector **A** is known in terms of its magnitude and the angle θ with the x -axis, its x and y components can be determined.

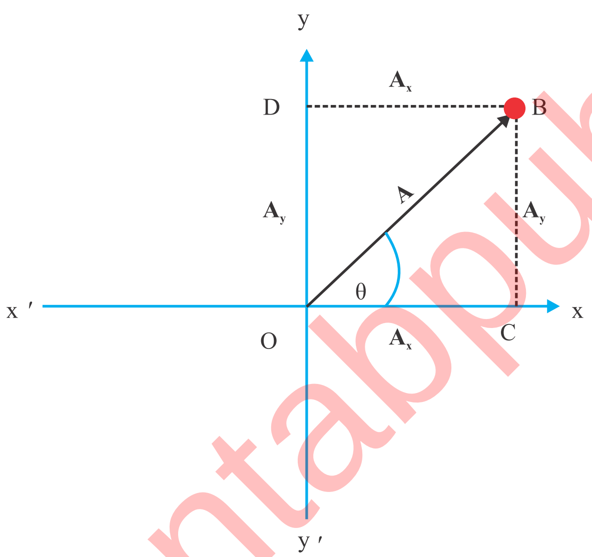


Figure 2.10: Resolution of a vector

Determination of vector from its rectangular components

If rectangular components A_x and A_y are known, the vector **A** can be determined by two methods. Either graphically, by head to tail rule, as discussed before or analytically.

To determine the vector from its components analytically, let us consider OCB in the Fig. 2.10. As it is a right-angled triangle, therefore, we can apply Pythagoras theorem to determine the hypotenuse 'A' because base (A_x) and perpendicular (A_y) are known.

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

Putting the values from the figure, we get:

$$A^2 = A_x^2 + A_y^2 \Rightarrow A = \sqrt{A_x^2 + A_y^2} \quad \text{----------}(2.4)$$

And knowing the perpendicular and base, the direction of the **A** can be determined by:

$$\tan\theta = \frac{\text{Perp}}{\text{Base}}$$

Putting the values from the figure, we get:

$$\tan\theta = \left(\frac{A_y}{A_x}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad \text{------}(2.5)$$

Thus, knowing the rectangular components, the vector and its direction can be determined with help of equation (2.4) and (2.5).

Determination of direction of a vector in different cases

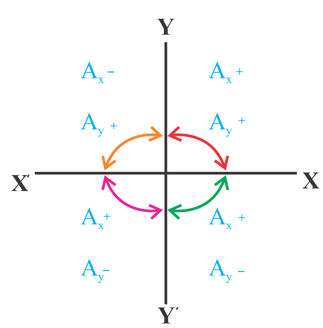
To determine the direction of a vector relative to the positive x-axis (measured counterclockwise), follow these steps:

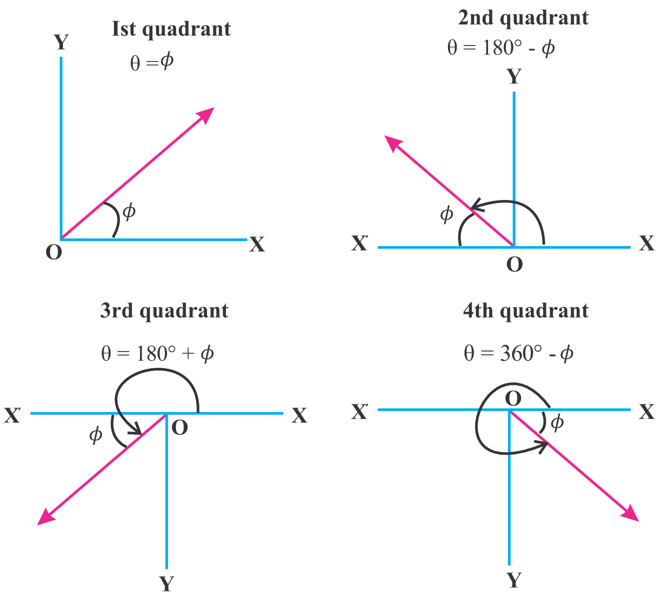
1 Calculate the reference angle $\phi = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right)$

This angle is computed without considering the signs of A_x and A_y.

2. Determine the quadrant of the vector using the signs of A_x and A_y

- First Quadrant: If $A_x > 0$ and $A_y > 0$, then $\theta = \phi$.
- Second Quadrant: If $A_x < 0$ and $A_y > 0$, then $\theta = 180° - \phi$.
- Third Quadrant: If $A_x < 0$ and $A_y < 0$, then $\theta = 180° + \phi$.
- Fourth Quadrant: If $A_x > 0$ and $A_y < 0$, then $\theta = 360° - \phi$.





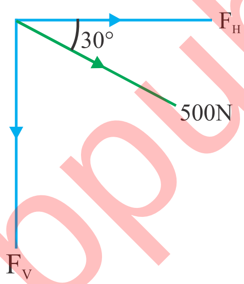
Assignment

An aircraft initiates the launch of a glider using a tow cable. At a specific instant, the cable's tension is 500 N, and it forms a 30° angle with the horizontal plane.

Determine:

a. The horizontal component of the force on the glider.

b. The upward force applied by the cable to the glider's front end.



"Launching Dynamics: A Glider Towed by an Aircraft at a 30° Angle with a Tension Force of 500 N".

Example 2.1

Find the magnitude and direction of the vector represented by each of the following pair of components?

a. $A_x = 5.60$ cm, $A_y = -8.20$ cm

b. $A_x = -2.70$ m, $A_y = -9.45$ m

c. $A_x = -3.75$ km, $A_y = 6.70$ km

Solution

(a) $A_x = 5.60$ cm , $A_y = -8.20$ cm

We know that the magnitude of a vector is:

$$A = \sqrt{A_x^2 + A_y^2}$$

and direction $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

So,

$$\sqrt{(5.60)^2 + (8.20)^2} = 9.93\text{cm} \quad \text{and}$$

$$\theta = \tan^{-1}\left(\frac{8.20}{5.60}\right) = 55.6°$$

This angle is not the actual angle because A_x is positive and A_y is negative. As vector lies in 4th quadrant. So the actual angle θ = 360°- ϕ = 360°-55.6°= 304.3°. It is the anticlockwise angle with +x-axis.

(b) $A_x = -2.70$ m , $A_y = -9.45$ m

$$A = \sqrt{(2.70)^2 + (9.45)^2} = 9.83 \Rightarrow A = 9.83\text{m}$$

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$ $\theta = \tan^{-1}\left(\frac{9.45}{2.70}\right) = 74.1$

θ = 74.1° which is not the actual, because A_x and A_y both are negative, so vector A is in third quadrant.

For third quadrant, we know that

θ = 180° + ϕ = 180° + 74.1° = 254° with +x-axis

(c) $A_x = -3.75$ km , $A_y = 6.70$ km

$$A = \sqrt{(3.75) + (6.70)^2} = 7.68 \text{ km}$$

and $\theta = \tan^{-1}\left(\frac{6.71}{3.75}\right) = 60.8°$

As A_x is negative and A_y is negative, so it is a second quadrant, so: θ= 180° – ϕ = 180° – 60.8° = 119.2° with +x-axis

Position vector

A vector that leads from the origin of a given coordinate system to a given point in space is called position vector. In a Cartesian coordinate system, a point P (x, y) is represented with respect to origin O (0,0) by a position vector **OP** shown in Fig. 2.9. There is a proper notation for a position vector i.e. **r**.

$$\overrightarrow{OP} = \mathbf{r} = \mathbf{x} + \mathbf{y} \Rightarrow \mathbf{r} = x\hat{i} + y\hat{j}$$

and its magnitude is $r = \sqrt{x^2 + y^2}$

In three dimensional space ⇒

$$\overrightarrow{OP} = \mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z} \qquad \mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

and magnitude of **r** is given by

$$r = \sqrt{x^2 + y^2 + z^2}$$

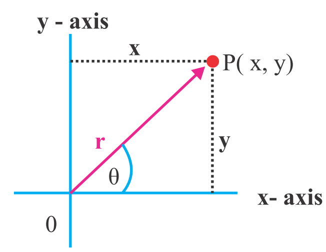


Figure 2.9: A position vector for a point P

Example 2.2

A cockroach is creeping on the floor of the kitchen. Selecting one corner of the kitchen as an origin of the rectangular coordinate system, if the coordinates of the cockroach are (2, 1, 0) in units of meter. Find the distance of the insect from the corner of the kitchen?

Solution

Coordinate of the position of cockroach: (x,y,z) = (2,1,0)
Corner of kitchen at origin of coordinate system = (0, 0, 0)
Distance from corner=?
The formula for the distance is: $d = \sqrt{x^2 + y^2 + z^2}$
Plugging in the coordinates of our cockroach, (x, y, z)= (2, 1, 0) we find:

$$d = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{4 + 1 + 0} = \sqrt{5} = 2.2\text{m}$$

Assignment

In a physics lab experiment, you are given two vectors:
$\mathbf{A} = 4\hat{i} + 3\hat{j}$ and $\mathbf{B} = -2\hat{i} + 6\hat{j}$ (where $\hat{i}$ and $\hat{j}$ are unit vectors along the x and y axes, respectively).

Determine the resultant vector R when vectors **A** and **B** are added. Also, calculate the magnitude and direction of the resultant vector **R**.

Multiple Choice Questions

1. If a vector **A** has a magnitude of 10 units and makes an angle of 30° with the positive x-axis, what is the x-component of this vector?
   a. 10 Cos30° b. 10Sin30°
   c. 5Cos30° d. 5Sin30°
2. Two vectors **A** and **B** have rectangular components $\mathbf{A} = 3\hat{i} + 4\hat{j}$ and $\mathbf{B} = 4\hat{i} - 3\hat{j}$. What is vector sum **A** + **B**?
   a. $7\hat{i} + \hat{j}$. b. $\hat{i} + 7\hat{j}$.
   c. $-\hat{i} + 7\hat{j}$. d. $7\hat{i} - \hat{j}$.
3. A vector **C** is defined as $\mathbf{C} = -2\hat{i} + 6\hat{j}$. What is the magnitude of **C**?
   a. 4.21 units b. 6.32 units
   c. 8.12 units d. 42 units

Test Yourself

1. Calculate the magnitude of the resultant force when two forces, 40 N due east and 30 N due north, act on an object.
2. A vector has a magnitude of 10 units and makes an angle of 45° with the positive y-axis. Find its x component.
3. Determine the direction of the vector, $\mathbf{A} = 4\hat{i} - 3\hat{j}$ with respect to the positive x-axis.

Knowledge 2.3 | Product of Two Vectors

As we know that when scalar quantities of different dimensions are multiplied, the dimensions of the product can possibly differ from either of the quantities being multiplied. For instance, distance covered by a moving body is the product of its speed and time interval. Similarly, when vectors are multiplied, product can have new physical dimensions. However, the rules for vector multiplication are different from that of the rules used for scalar multiplication. *The product of two vectors can yield either a scalar quantity or a vector quantity.*

Scalar Product

When two vectors are multiplied and the result is a scalar quantity, the product is called scalar product. The notation **A . B** read as dot product of **A** and **B** or simply **A** dot **B**. Mathematically, it can be written as:

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta \quad \text{---------------} \quad (2.6)$$

where A and B are the magnitudes of **A** and **B**, θ is the angle between **A** and **B** as indicated . Since A and B are scalars and cos θ is a number, the scalar or dot product of two vector quantities is a scalar quantity. Dot product of two vectors can be regarded as the product of the magnitude of one vector and the component of the other vector in the direction of the first vector, thus, can correspondingly be expressed either as

A . B = A(B cos θ) = magnitude of **A** [projection of **B** on **A**]

or

B. A = B(Acos θ) = magnitude of **B** [projection of **A** on **B**]

This is illustrated in Fig. 2.11.

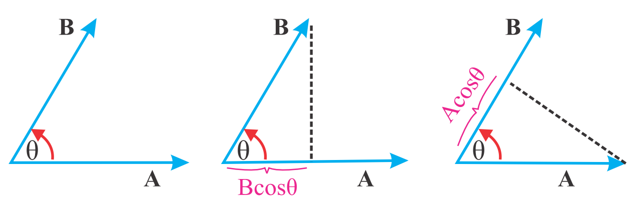


Figure 2.11: Scalar product of two vectors

For example; work is defined as the product of the effective force and distance. Effective force is the component of the force in the direction of displacement. When force **F** and displacement d are in different directions as indicated in Fig.2.12, the effective force is the component of **F** in the direction of **d** which is (Fcos θ). So

$W = (F \cos \theta)\, d$

$W = (Fd \cos \theta)$

$W = \mathbf{F} \cdot \mathbf{d}$

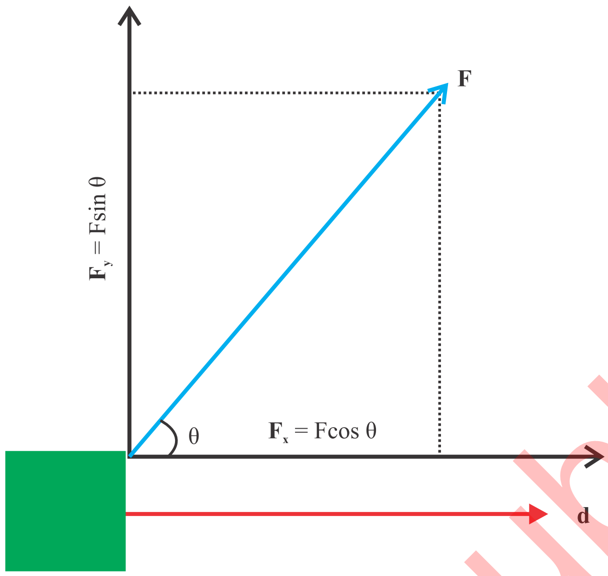


Figure 2.12: Illustrating work as the Dot Product of Force and Displacement in Vector Components."

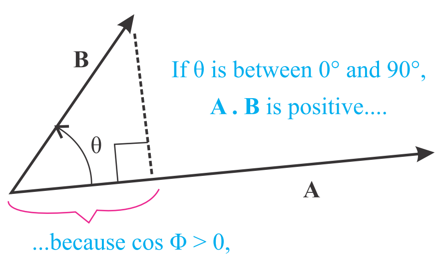
Several important physical quantities can be described in the same way.

- Power = (force) dot (velocity) = $\mathbf{F} \cdot \mathbf{v}$.
- Electric flux = (electric intensity) dot (vector area) = $\mathbf{E} \cdot \mathbf{A}$.
- Magnetic flux = (magnetic flux density vector) dot (vector area) = $\mathbf{B} \cdot \mathbf{A}$.

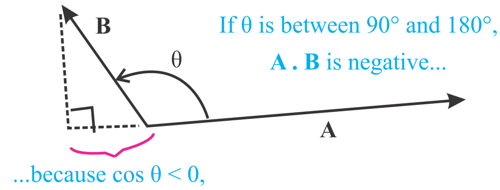
Key Facts

- The scalar product $\mathbf{A} \cdot \mathbf{B}$ = A B cos θ can be positive, negative, or zero, depending on the angle between **A** and **B**.

(a)



(b)



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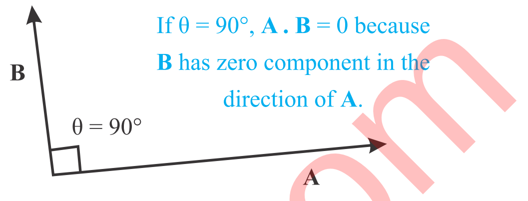


Characteristics of Scalar Product

1. Using the definition of scalar product in Eq 2.6, A, B and cos θ are real numbers and they commute, therefore, we can say that scalar product obeys commutative law, i.e. the order of multiplication does not affect the result,

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}.$$

2. If **A** and **B** are the two non-zero vectors then depending on the angle between them vectors, we can discuss some special cases:

i. θ = 0°, which means that given vectors are parallel to each other. In this case the dot product will be equal to the product of magnitude of the given vectors:

$$\mathbf{A} \cdot \mathbf{B} = AB \qquad \cos \theta = 1$$

It is the maximum value of scalar product.

ii. The **self product** of two vector is equal to the square of magnitude of vector.

$$\mathbf{A} \cdot \mathbf{A} = A A \cos 0° = A^2$$

In terms of Cartesian unit vectors, we can write:

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1 \qquad \because \ |\hat{i}|^2 = 1$$

iii. If θ = 180°, it means that given vectors are antiparallel to each other. Since, cos180 = -1, in such case, we can write:

$$\mathbf{A} \cdot \mathbf{B} = -AB$$

and $\hat{i} \cdot (-\hat{i}) = \hat{j} \cdot (-\hat{j}) = \hat{k} \cdot (-\hat{k}) = -1$

iv. If θ = 90°, which means that given vectors are perpendicular to each other. In this case, the product will vanish, since cos 90 = 0 :

$$\mathbf{A} \cdot \mathbf{B} = 0$$

Hence two non-zero vectors are orthogonal if their dot product is zero.

In terms of Cartesian unit vectors, we can write:

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

3. If **A**, **B** and **C** are the three vectors, then:

$$\mathbf{A} + (\mathbf{B}\,.\,\mathbf{C}) = \mathbf{A}\,.\,\mathbf{B} + \mathbf{A}\,.\,\mathbf{C}$$

This shows that scalar product is distributive over addition.

4. From the special cases discussed above, the product of two vectors in terms of rectangular components $\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\mathbf{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ can be written as:

$$\mathbf{A}\,.\,\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

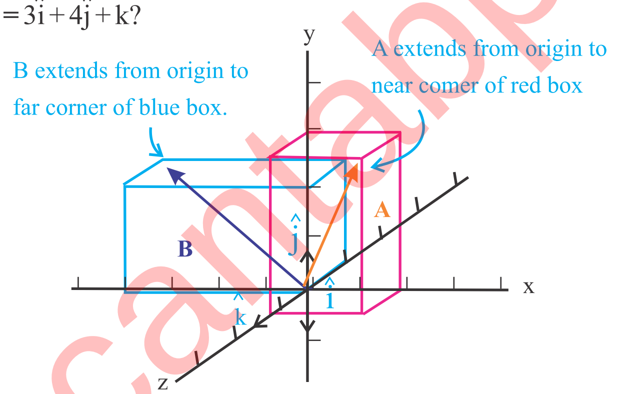
5. Angle between the vectors **A** and **B** can be determined as

$AB\cos\theta = A_x B_x + A_y B_y + A_zB_z$

$$\theta = \cos^{-1}\left(\frac{A_x B_x + A_y B_y + A_z B_z}{AB}\right)$$

Assignment

You have two vectors $\mathbf{A} = 7\hat{i} + 3\hat{j} - 2\hat{k}$ and $\mathbf{B} = 4\hat{i} - \hat{j} + 5\hat{k}$ in three-dimensional space. Calculate the dot product of these vectors. Based on the result, comment on the relative orientation of these vectors. Further, if these vectors represent forces, what condition must be true about their work on an object moving along the displacement vector **C** $= 3\hat{i} + 4\hat{j} + \hat{k}$?



Two vectors in three dimensions

Example 2.3

A cart is pulled a distance of 50 m along a horizontal path by a constant force of 25 N. The handle of the cart is pulled at an angle of 60° above the horizontal. Find the work done by the force.

Solution: F = 25 N, d = 50 m, θ = 60°

$W = F.d = Fd\cos\theta$

$W = (25)(50)\cos 60°$

$W = (1250)(0.5) = 625\,J$

Example 2.4

Find the angle between the vectors

$\mathbf{A} = 2.00\hat{i} + 3.00\hat{j} + 1.00\hat{k}$

and

$\mathbf{B} = -4.00\hat{i} + 2.00\hat{j} - 1.00\hat{k}$

Solution

Angle between two vectors A and B is determined as

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

Here we have $A_x = 2.00$, $A_y = 3.00$, and $A_z = 1.00$, and

$B_x = -4.00$, $B_y = 2.00$, and $B_z = -1.00$. Thus

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (2.00)(-4.00) + (3.00)(2.00) + (1.00)(-1.00)$$
$$= -3.00$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2.00)^2 + (3.00)^2 + (1.00)^2}$$
$$= \sqrt{14.00}$$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4.00)^2 + (2.00)^2 + (-1.00)^2}$$
$$= \sqrt{21.00}$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3.00}{\sqrt{14.00}\sqrt{21.00}} = -0.175$$

$$\theta = 100°$$

Vector Product or Cross Product

When multiplication of two vectors yields another vector, then the product is called vector or cross product. The notation **A** × **B** read as vector or cross product of **A** and **B** or simply **A** cross **B**. Mathematically, it can be written as:

$$\mathbf{A} \times \mathbf{B} = \mathbf{C} = AB\sin\theta\,\hat{n} \qquad \text{---------} \quad (2.7)$$

where A and B are the magnitudes of A and B and θ is the smaller angle between **A** and **B**. The unit vector $\hat{n}$ gives the direction of the **A × B**. The cross product **A** × **B** is perpendicular to the plane containing both the vectors **A** and **B**. The magnitude of the product vector **C** ($C = AB\sin\theta$) is equal to the area of the parallelogram that the two vectors span.

The direction of **C** is given by the right-hand Fleming rule, as shown in Fig .2.13, where *one simply points the forefinger of the right hand in the direction of **A** and the middle finger in the direction of **B**. Thumb provides the direction of the product vector **C** represented by unit vector n, which is always perpendicular to the plane containing **A** and **B**.*

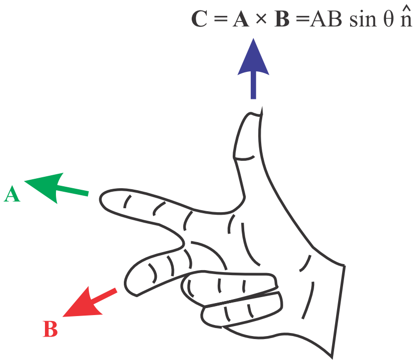
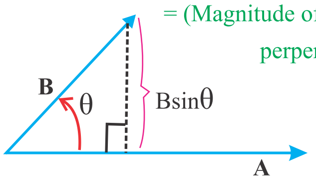


Figure 2.13: Right hand rule

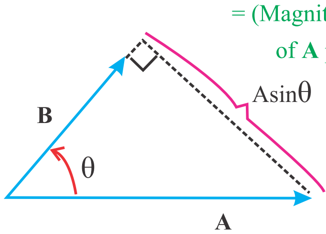
Key Facts

- Calculating the magnitude AB sin θ of the vector product of two vectors, **A** × **B**

(a) Magnitude of **A** x **B** equals A(Bsinθ).
= (Magnitude of **A**) x (Component of **B** perpendicular to **A**)



(b) Magnitude of **A** x **B** also equals B(Asinθ).
= (Magnitude of **B**) x Component of **A** perpendicular to **B**)



Some common examples of vector product are ;

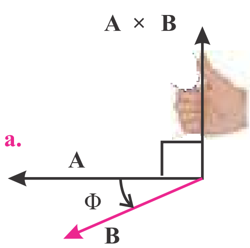
- Torque = (position vector relative to pivot point) × (force) = **r** × **F**.
- Angular momentum = (position vector) × (linear momentum) = **r** × **P.**
- Magnetic force = (charge) (velocity × magnetic flux density vector) = q(**v** × **B**).

Key Facts

- Alternate method to determine the direction of (a) **A** × **B** and (b) **B** × **A**

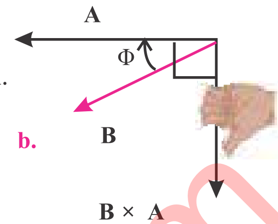
(a) Using the right-hand rule to find the direction of **A** × **B**

- Place **A** and **B** tail to tail.
- Point fingers of right hand along **A**, with palm facing **B**.
- Curl fingers toward **B**.
- Thumb points in the direction of **A** x **B**



(b) Using the right-hand rule to find the direction of **B** × **A** = **-A** × **B** (vector product is anticommutative)

- Place **B** and **A** tail to tail.
- Point fingers of right hand along **B**, with palm facing **A**.
- Curl fingers toward **A**.
- Thumb points in direction of **B x A** which has same magnitude as **A x B** but points in opposite direction.



Characteristics of cross product

1. Right-hand rule described in Fig. 2.13 tells us that, unlike scalar product, cross product is non-commutative, i.e; $\mathbf{A}\times\mathbf{B} \neq \mathbf{B}\times\mathbf{A}$ but $\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$.

2. If **A** and **B** are the two non-zero vectors then depending on the angle between the vectors, we can discuss some special cases:

i. If θ = 0°, which means that given vectors are parallel to each other. In this case the vector product vanishes, sin θ = 0: $\mathbf{A}\times\mathbf{B} = 0$

In terms of Cartesian unit vectors, we can write:

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

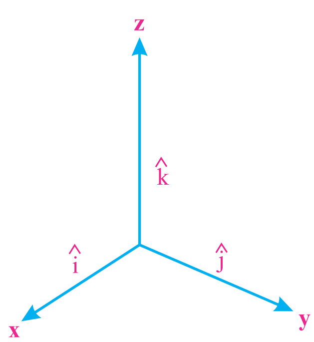
ii. If θ = 180°, which means that given vectors are antiparallel to each other. In this case, the vector product vanishes as well since sin 180° = 0:

iii. If θ = 90°, which means that given vectors are perpendicular to each other. In this case, the magnitude of the vector product will be maximum, since sin 90° = 1:

$$\mathbf{A}\times\mathbf{B} = AB\sin 90°\,\hat{n} = AB\,\hat{n}$$

In case of Cartesian unit vectors which are perpendicular to each other, we can write:

$$\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$$



$$\hat{j}\times\hat{i} = -\hat{k}$$
$$\hat{k}\times\hat{j} = -\hat{i}$$
$$\hat{i}\times\hat{k} = -\hat{j}$$

Figure 2.14: A right-handed coordinate system

3. From these special cases discussed above, we can find the product of two vectors $\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\mathbf{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ in terms of rectangular components as: $\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$. This product can also be expressed in terms of a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

4. Like scalar product, vector product is also distributive over addition. If **A, B** and **C** are three vectors, then:

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

This is known as the distributive property of cross product over addition.

5. The magnitude of **A×B** equals the area of the parallelogram formed by vectors **A** and **B** as two adjacent sides, as depicted in the Fig. 2.15.

Area of parallelogram OPQR

= (base) (Altitude)

= (OP) (QM)

= (A) (B sin θ)

=AB sin θ

$= |\mathbf{A} \times \mathbf{B}|$

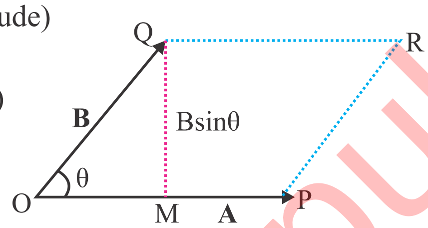


Figure 2.15

Assignment

Consider two vectors in three-dimensional space, $\mathbf{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\mathbf{B} = \hat{i} - 2\hat{j} + 4\hat{k}$. Calculate the cross product A x B. Use this result to determine the area of the parallelogram formed by vectors **A** and **B**. Additionally, if a third vector **X** $= 5\hat{i} + 4\hat{j} + 3\hat{k}$ lies in the plane of this parallelogram, what can you infer about the scalar product of **A.B** with vector **X**?

Example 2.5

Given two vectors $\mathbf{A} = 3\hat{i}\ (-2\hat{j}) + \hat{k}$ and $\mathbf{B} = -\hat{i} + 4\hat{j}$, find the cross product **A × B**.

Solution

To find the cross product, use the determinant method:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ -1 & 4 & 0 \end{vmatrix}$$

Expanding this determinant:

$$\mathbf{A} \times \mathbf{B} = \hat{i}((-2)(0) - (1)(4)) - \hat{j}((3)(0) - (1)(-1)) + \hat{k}((3)(4) - (-2)(-1))$$

Simplify each component:

$$\mathbf{A} \times \mathbf{B} = \hat{i}(-4) - \hat{j}(1) + \hat{k}(10)$$

Thus, the cross product is:

$$\mathbf{A} \times \mathbf{B} = -4\hat{i} - \hat{j} + 10\hat{k}$$

Example 2.6

Magnitude: **A × B** is 3/4 and magnitude of **A . B** is 1/2. Find the angle between the vectors A and B.

Solution

$$|\mathbf{A} \times \mathbf{B}| = |A||B|\sin(\theta) = \frac{3}{4} - - - - - - - - (1)$$

$$\mathbf{A} \cdot \mathbf{B} = |A||B|\cos(\theta) = \frac{1}{2} - - - - - - - - -(2)$$

Dividing eq(1) by eq(2)

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \times 2 = \frac{3}{2}$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3^{o}$$

Multiple Choice Questions

1. What is the dot product of two perpendicular vectors $\vec{A}$ and **B**?
   a. Equal to the product of their magnitudes. b. Zero.
   c. Equal to the magnitude of one of the vectors.
   d. Cannot be determined without additional information.
2. If $\mathbf{A} = (3\hat{i} + 4\hat{j})$ and $\mathbf{B} = (4\hat{i} - 3\hat{j})$, what is the dot product **A . B**?
   a. 0 b. 7 c. 25 d. –25

Test Yourself

1. State the significance of the dot product of two vectors.
2. Find the area of the parallelogram formed by vectors $\mathbf{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\mathbf{B} = -\hat{i} + 2\hat{j} + \hat{k}$

Skill 2.1

- Differentiate between scalar and vector quantities, accurately represent vectors in two-dimensional space, and perform and interpret the results of vector operations, specifically dot and cross products.

Key Points

- Scalars are quantities defined only by magnitude (like mass or temperature), whereas vectors have both magnitude and direction (like force or velocity). This distinction is crucial in physics and engineering for accurate descriptions of physical phenomena.
- Vector representation involves illustrating physical quantities that have both magnitude and direction, using notations like bold letters, arrows, or line segments with arrowheads. This visual representation aids in understanding and calculating vector interactions.
- Cartesian coordinate system uses perpendicular axes (typically labeled x, y, and z for 3D) to describe the position of points in space. It is fundamental in mathematics and engineering for plotting points, lines, and shapes in a plane or space.
- Vectors can be broken down into their constituent parts (components) along the axes of a coordinate system and the algebraic rules governing vector addition, subtraction, and multiplication.
- Graphical and analytical methods involve visually depicting vectors and using mathematical formulas for operations like addition and subtraction. Graphical methods provide intuitive understanding, while analytical methods offer precision.
- Explores the characteristics of vectors, including how they behave under operations like scaling (multiplication by a scalar) and how properties like direction and magnitude change under these operations.
- Vector resolution involves breaking down a vector into its orthogonal components, typically along the x and y axes. This is essential for simplifying complex vector calculations and analyses.
  Scalar (dot) product results in a scalar and is useful in calculating quantities like work. The vector (cross) product results in a vector, relevent in determining quantities like torque.
- The cross product of two vectors has unique characteristics, such as the right-hand rule for determining the direction of the resultant vector and the anti-commutative nature of the cross product, where reversing the order of the vectors change the sign of the result.

Exercise

A Multiple Choice Questions:

Select the best answer of the following questions.

1. Two unequals, unlike parallel forces acting simultaneously along different lines of action may produce

a. Translatory motion on
b. Rotatory motion only
c. Translatory and rotatory motion
d. No motion

2. If vector **B** is added to vector **A**, under what condition does the resultant vector **A + B** has a magnitude A + B, when the angle between the two vectors is:

a. 0° b. 45° c. 90° d. 180°

3. If **A + B = 0**, the corresponding components of the two vectors **A** and **B** must be:

a. Equal b. positive c. negative d. opposite sign

4. The status of vector **B**, if vector **A** = 0 and **A + B = A – B**, is:

a. |**A**|= 2|**B**| b. |**A**|= |**B**| c. **B = 0** d. **A⊥B**

5. For which angle the equation |**A . B**| = |**A × B**| is correct?

a. 30° b. 90° c. 45° d. 60°

6. Two forces of magnitude 20 N and 10 N acts at a point then which of the following cannot be their possible sum?
a. 10 N b. 15 N c. 30 N d. 35 N

7. When is the dot product of two vectors **A** and **B** equal to zero?
a. When the vectors are parallel to each other.
b. When the vectors are antiparallel to each other.
c. When the vectors are perpendicular to each other.
d. The dot product is never equal to zero.

8. What is the direction of the cross product **A** × **B** when vectors **A** and **B** are perpendicular to each other?
a. It is parallel to vector **A**.
b. It is parallel to vector **B**.
c. It is perpendicular to both vectors **A** and **B**.
d. It is in the opposite direction of vector **A**.

9. To fly due north while the airplane's airspeed is 180 km h^{-1} and the wind blows from the west at 60 km h^{-1}, determine the direction the pilot must steer the airplane.
a. 18° east of north
b. 18° west of north
c. 19.5° east of north
d. 19.5° west of north

10. What is the angle formed by the vector $-2\hat{i} + 2\hat{j}$ with the positive x-axis?
a. 45°
b. 135°
c. 225°
d. 315°

11. What is the angle between the vectors $3\hat{i} - 4\hat{j} + 5\hat{k}$ and $-2\hat{i} + 3\hat{j} - 6\hat{k}$?
a. 0°
b. 60°
c. 90°
d. 180°

12. What is the angle between the vectors $2\hat{i} - 4\hat{j} + 6\hat{k}$ and $-\hat{i} + 2\hat{j} - 3\hat{k}$?
a. 0°
b. 60°
c. 90°
d. 180°

13. If the dot product of two non-zero vectors is zero, what can be said about the vectors?
a) They are parallel.
b) They are perpendicular.
c) They have the same magnitude.
d) They lie on the same plane.

14. The cross product of two vectors results in a vector that is:
a) Parallel to and.
b) Perpendicular to both and .
c) In the same direction as .
d) In the same direction as .

15. Which property does not apply to the cross product of two vectors ?
a) Distributive over vector addition.
b) Commutative.
c) Anticommutative.
d) Produces a vector orthogonal to both .

B Assertion – Reason Type Questions

Answer the assertion-reason type questions from a given pattern in each of the following questions. Two statements are labeled as assertion (A) and other labeled as a reason in (R) are shown. examine both the statements and mark the correct choice according to the instructions given below (a) if both A & R are correct & R is the reason for A (b) if both A & R are correct, but R is not the reason for A (c) If A is Correct and R is wrong (d) if A is wrong and R is correct

1. **A:** The magnitude of a vector remains constant regardless of the coordinate system used.
R: The magnitude of a vector is a scalar quantity.

2. **A:** In a Cartesian coordinate system, a vector can be resolved into its x and y components.
R: The resolution of a vector into components depends on the chosen coordinate system.

3. **A:** The scalar product of two vectors is a vector quantity.
R: Scalar products combine the magnitudes of two vectors and the cosine of the angle between them.

4. **A:** The cross product of two vectors is always perpendicular to the plane containing the vectors.
R: The direction of the cross product is determined by the right-hand rule.

5. **A:** In vector addition, the commutative law always applies.
   R: The order of adding vectors does not change the resultant vector.
6. **A:** A unit vector has a magnitude other than one.
   R: Unit vectors are used to indicate direction only.
7. **A:** The dot product of two perpendicular vectors is non-zero.
   R: Perpendicular vectors have an angle of 90 degrees between them.
8. **A:** Parallel vectors have the same direction but may differ in magnitude.
   R: Parallel vectors are scalar multiples of each other.
9. **A:** The resultant of two vectors using the parallelogram law is the diagonal of the parallelogram.
   R: The parallelogram law applies to vectors represented by adjacent sides of a parallelogram.
10. **A:** Negative vectors have the same magnitude as their corresponding vectors but opposite direction.
    R: A negative vector is obtained by multiplying the vector by -1.
11. **A:** Scalar multiplication changes the direction of a vector.
    R: Scalar multiplication only affects the magnitude of a vector.
12. **A:** The components of a vector are independent of each other.
    R: A vector can be represented as the sum of its components.
13. **A:** Two vectors having the same magnitude and direction are equal.
    R: Equal vectors are identical in all aspects, including their points of application.
14. **A:** The cross product of two parallel vectors is zero.
    R: Parallel vectors have an angle of 0 degrees or 180 degrees between them.
15. **A:** The work done by a force is the dot product of force and displacement vectors.
    R: Work is a scalar quantity representing the product of force magnitude and displacement magnitude in the direction of the force.

C Restricted Response Question

2.1 What happens to the direction of a vector if its components are swapped?

2.2 Can the scalar product of two vectors be negative? If the answer is yes, provide a proof and give an example.

2.3 Suppose **A** is a non-zero vector. If **A**.**B** = 0 and **A** x **B** = 0, where **B** is an unknown vector. What can you conclude about **B**?

2.4 You must have seen trucks and buses with larger steering wheel. Explain it with reference to the force exerted by the driver while turning the vehicle.

2.5 How would you decompose a force vector into components that are parallel and perpendicular to an inclined plane?

2.6 In certain cases, even though neither the force (**F**) nor the displacement (**S**) is zero, the work done (W = **F**. **S**) is zero. What conclusions can be drawn from this situation?

2.7 How the perpendicular components of a force vector would change if the direction of the force is altered by 30°?

2.8 The wind is blowing from the south at a speed of 15 m/s. However, a cyclist perceives the wind as coming from the east at the same speed of 15 m/s. What is the velocity of the cyclist?

2.9 Identify the conditions where **A** x **B** = 0?

2.10 Is it possible to add zero to the null vector?

2.11 Analyze the effect of doubling a vector's magnitude on its perpendicular components.

2.12 What is the result of the dot product when two vectors are orthogonal

D Extended Questions

2.1 Write the steps involved in splitting a vector into its rectangular components? Also comment about the magnitude of x- and y-component with the variation of the angle, the vector which it makes with x-axis.

2.2 What is scalar product of vectors? Discuss its characteristics.

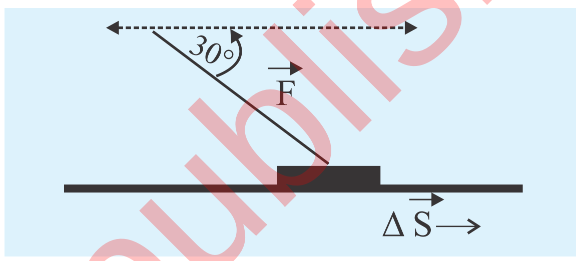
2.3 What is cross product? Discuss its physical significance. Also write its three characteristics

E Numerical Problem

Problem 2.1 What is the displacement of the point of a wheel initially in contact with the ground when the wheel rolls forward one-half revolution; Radius of the wheel is R? **[3.72 R and 32°]**

Problem 2.2 Resultant of two forces equal in magnitude, at right angles to each other is 1414 N. Find the magnitude of each force. **[999.8 N]**

Problem 2.3 A force of 20 N acts on a body and moves it 2 m. The line of motion of the body and direction of the force are shown in the figure. Calculate the work done. **[34.6 J]**



Problem 2.4 Vectors **A**, **B** and **C** 4 units north, 3 units west and 8 units east respectively.
Find **a.** **A** x **B** **b.** **A** x **C** **c.** **B** x **C**

[12 unit vertically up, 32 unit vertically down, 0 unit]

Problem 2.5 Two vectors represent electric force and displacement acting on a charge of 1C : **F** = $4\hat{i}+3\hat{j}$ N and **d** = $5\hat{i}-2\hat{j}$ m. Calculate the work done in moving 1C charge aong the displacement.

[14 J]

Problem 2.6 A force of 10N is applied to a wrench at an angle of 60° to the lever arm, which is 0.5m long. Calculate the magnitude of the torque applied to the wrench. **[4.33 N m]**

Problem 2.7 Given a force with one of its rectangular components measuring 40 N and forming an angle of 45° with the force, calculate the magnitude of the other component. **[40N]**

Problem 2.8 A worker pushes a heavy crate along a warehouse floor with a force vector **F** = $100\hat{\imath} + 60\hat{\jmath}$ N. The crate moves in the direction of a displacement vector **d** = $4\hat{\imath} + 3\hat{\jmath}$ m. Calculate the projection of the force vector **F** onto the displacement vector **d** **[116 N]**

Problem 2.9 Determine the relative velocity of the dog with respect to the monkey as the monkey climbs a vertical pole at 8 m s^{-1} while the dog approaches the pole at 12 m s^{-1}.

[14.4 m s^{-1}]

Problem 2.10 Calculate the displacement of a person from their initial position after moving 70 m north, followed by 50 m east, and finally $60\sqrt{2}$ m south-west. **[14.1 m]**

Problem 2.11 Two vectors A and B have a dot product of 12 and a cross product magnitude of 8. Find the angle between these two vectors. **[53.1°]**

Problem 2.12 A force $F = 4\hat{i}-2\hat{j}+3\hat{k}$ N is applied at a point with position vector $r = 5\hat{i}+ 2\hat{j}-3\hat{k}$ m.

(a) Calculate the torque τ about the origin.

(b) If the point of application of the force moves to a new position given by the vector $r' = 2\hat{i}+3\hat{j}+\hat{k}$, what is the new torque about the origin?

[$-27\hat{j}-18\hat{k}$ Nm, $11\hat{i}-2\hat{j}-16\hat{k}$ Nm]